Unit 8: The Electric Potential

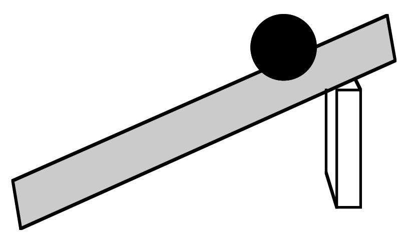
This unit introduces the concept of the electric potential. The electric potential bears the same relationship to the electric field that potential energy bears to force. While this seems like a new concept, we actually have spent a great deal of time dealing with this in a practical sense--it is the voltage we used in electric circuits.

Session 1: Potential Energy and Electric Potential

Before we investigate the electric potential, it will be helpful to remind ourselves of the mechanical concepts of force and potential energy.

Guidebook Entry VIII.1: Bowling on the Incline

To allow us to be concrete in our discussion of force and potential energy, we will consider a ball rolling down an inclined plane. To help guide your investigation, you have a bowling ball and a smooth board to test your intuition.



On the figure below, which represents the surface of the plane, place arrows that indicate the direction and magnitude (qualitatively) of the net force that the ball experiences on the plane.

Recall that the potential energy associated with gravity is given by $U = mgh$, where h is the vertical height of the object of mass m. Draw a line (curved or straight) on the board which corresponds to all the points at which the ball would have the same value of the potential energy as it did at the very center of the board. (If you wish, you may use a ruler and a marker to help you with the real board.) Mark this line with the letter A. How does this line relate to the force directions?

Draw a line on the board that corresponds to some constant potential energy a bit (call it ΔU) greater than line A. Mark this as line B. Then draw a line that corresponds to ΔU less than the line A, and mark this as line C. (Again, feel free to use a marker and a ruler.)

Describe in a few sentences the direction of the force lines, the direction and spacing of the equi-potential energy lines, and the relationship between the two sets of lines.

What if the board had some serious warpage and was twisted up, or the ball was rolling down a hillside? Describe in a few sentences the direction of the force lines, the direction and spacing of the equi-potential energy lines, and the relationship between the two sets of lines. Discuss this with your partners, and then discuss your answers with an instructor.

When the concept of work was first introduced, it was introduced in a one dimensional setting, where work done by a force was the product of force times distance traveled, or for a force that was not constant,

$$W = \int F \cdot dx \quad .$$

For forces in two and three dimensions, we need to take into account the direction of the force relative to the direction of travel, as expressed by the dot product in the expression

$$W = \int \vec{F} \cdot d\vec{x} = \int F \cos\theta \, dx$$

where θ is the angle in between the force direction and the direction of travel. When the work done in getting from point A to point B didn't depend at all on how you got between those two points, then we were able to define a potential energy U which is a function of position such that the change in U is given by

$$\Delta U = -W$$

Using this, can you make sense of the geometric relationship between the equi-potential energy lines and the lines of force? To help you think about this, consider the following. If the work done comes from the potential energy of the object, how much work is done in moving along an equi-potential energy line?

We can also turn this integral relationship around, and see that the force that is exerted is equal to the derivative of the potential energy function. In one dimension, this is a simple derivative. In two or more dimensions, this is a vector function called the gradient:

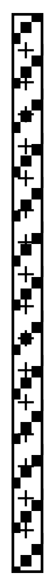
$$\vec{F} = -\vec{\nabla} U = -\left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right) .$$

For our analogy of a ball rolling on a hillside, the force vector is in the direction of steepest descent, and is equal to the rate of change of height versus horizontal distance in the direction of steepest descent.

Next, we will consider these same ideas for a case of constant *electric* force.

Guidebook Entry VIII.2: Forces and Potential Energy Near a Charged Plate

In the last couple of sessions, we have considered the electric field due to an infinite charged flat surface. Imagine such a surface oriented in the vertical direction, as shown.



Using arrows, sketch on the previous figure the forces that a positive test charge would feel at various points around the plate, including (qualitatively) both direction and magnitude. Explain any reasoning below.

Sketch on the figure the lines corresponding to equal potential energy surfaces at equally spaced values (e.g. 1 Joule, 2 Joules, 3 Joules ...). Explain your reasoning below.

What quantity would be represented by the electric force divided by the magnitude of the test charge?

Just as we found it convenient to define the electric field as the force that a test charge would feel at any given point in space divided by the magnitude of the test charge, we can now define the electric potential (symbolized by V) to be the potential energy that the test charge would have at that point divided by the magnitude of the test charge. All of the relationships that we reviewed above for forces and potential energy apply to the case of electric fields and the electric potential:

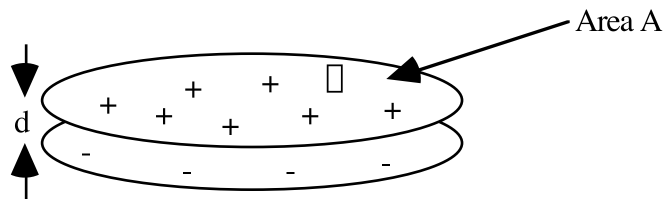
$$V = -\int \vec{E} \cdot d\vec{x} = -\int E \cos\theta \, dx$$

and

$$\vec{E} = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right) .$$

Guidebook Entry VIII.3: Electric Potential in a Capacitor

Consider a capacitor that is constructed from two parallel plates. Let the plates be much larger than the spacing in between them, so that we may treat them as being virtually infinite. Let the plates each have an area of A, a spacing of d, and a surface charge density (one positive, the other negative) of σ.



What is the electric field in between the two plates in terms of σ and ϵ_0? You may wish to use Gauss's law ($E \cdot A = q_{enclosed}/\epsilon_0$, where $\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$) to find this one, or refer to your activity guide or to another reference.

Make an argument why this field must be uniform (constant in magnitude and direction).

Given the uniform electric field and spacing, what is the potential of the upper plate if the potential of the lower plate is zero? (What you are calculating here is the potential difference, or voltage, between the two plates.)

What is the total charge on the upper plate, given the area A and the charge density σ?

Calculate the ratio between the charge on one of the plates and the voltage between the two plates. What is another name for this ratio?

Guidebook Entry VIII.4: Forces in a Real Capacitor

Now is your chance to show you really have a handle on this stuff, and check it against reality! I have set up our capacitor on a balance again. Make whatever measurements of the dimensions of the apparatus that you need to, and then predict what the force should be on the lower plate when the voltage is turned up to 1000 V.

Now ask your instructor to turn on the power supply, and check your result! (The balance reads in grams. Don't forget that not only are grams *not* MKS units, they are not even force units!)

Session 2: Potential Energy and Electric Potential

In this session, we will look carefully at the relationship between electric potential (voltage), charge and current and how this relates to energy and power. These are notions that we have been playing with in a rather casual and incidental way, but now we will address these relationships directly and quantitatively.

Guidebook Entry VIII.5: A Capacitor Discharged Through a Bulb--Energy Conservation

What do you recall happens when you do the following experiment:
1) Connect a capacitor directly to a 3 V battery for a few (maybe ten) seconds, then
2) Disconnect the battery, and then connect a light bulb instead.

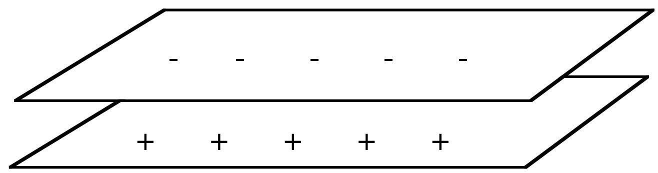
You may wish, especially if you are not absolutely certain what will happen, to perform this experiment. Are your results what you expected?

Through a variety of earlier courses, you have learned about conserveration of energy. In your own words, describe if and how energy might be conserved in this experiment. To be specific, what is the "kind" of energy at each stage, and where is it "stored." Discuss this thoroughly with your partners and an instructor.

You likely came to the conclusion that the energy that eventually came out as light and heat in the bulb came from the battery originally, but was somehow stored in the capacitor in between. Now we want to investigate more carefully how that energy gets stored in the capacitor.

Guidebook Entry VIII.6: Energy Changes in a Discharging Capacitor

Imagine a parallel plate capacitor that is charged, as shown.



Sketch in the direction of the electric field.

If we move a small amount of the positive charge from the bottom plate to the top plate, does this tend to charge or to discharge the capacitor? Explain.

Do you expect the energy stored in the capacitor to increase or decrease as you move the charge from the lower plate to the upper one? Explain.

The capacitor is charged to 1000 V, and the spacing between the two plates is 0.01 m. What is the electric field between the two plates? Recall that the electric field is the negative of the derivative of the potential:

$$E = -\frac{dV}{dx} \approx -\frac{\Delta V}{\Delta x}$$

where the last approximation is good when E is nearly a constant (it actually gives the average electric field over the region Δx).

The charge that we move from the lower plate to the top plate is is 1 pC (1 picocoulomb is 10^{-12} Coulombs), which is small compared to the total charge on the capacitor. How much work does this do on the outside (i.e. if it is analogous to a ball rolling downhill) or how much work is done on the charge (i.e. if it is analogous to a ball rolling uphill)? Calculate this from the force integrated over the distance.

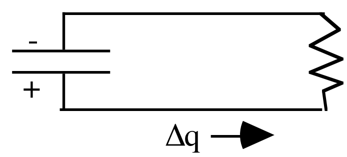
Recall that the voltage is the potential energy per unit charge that a small test charge experiences. Use the potential difference between the two plates to calculate the energy change in moving 1 pC from the lower plate to the upper. Does it agree with your previous answer, and with your qualitative description of the total energy change?

Now imagine that a capacitor charged to voltage V has some tiny amount of charge q moved from one plate to the other. How much does the energy change? We'll use this expression again, so check with your instructor!

In this last exercise, we assumed that we discharged a capacitor by moving the charge from one plate directly to the other. However, when we discharge a capacitor, we let some charge leak off through a wire and circuit elements over to the other plate. Since the electric force, like the gravity force, conserves energy, it doesn't matter how we get the charge from one plate to the other, the change in energy is the same. With this in mind, we'll apply this to a discharge through a resistor, and use this to understand the energy changes of an electric current passing through a resistor.

Guidebook Entry VIII.7: Current Passing Through a Resistor

Let's now move this charge from the bottom plate to the top by moving it through an external resistor.



If the energy stored in the capacitor decreases in this process by an amount ΔU, where does this energy go? Explain, and check with an instructor.

If this energy is dissipated over a period of time Δt, what is the power P = dU/dt (i.e. the time rate of energy loss) in terms of the charge moved (Δq) and the voltage difference V across the resistor?

If the charge Δq flows at a constant rate over the time Δt, what is the current I in terms of Δq and Δt?

Now express the power in terms of the current flowing through the resistor and the voltage across the resistor.

This expression for the power across a resistor, P = VI, actually had nothing to do with the fact that this was a resistor. It is also valid for other circuit elements as well. The one characteristic that is different for a resistor is that the energy is actually lost--turned into heat. For other circuit elements, such a capacitors, the energy is stored, and we can later recover it. In the next exercise, we will use the thermal nature of this energy loss to verify our power expression.

Guidebook Entry VIII.8: Verifying Heat Produced in a Resistor

In this exercise, we wish to make a resistor generate a considerable amount of heat. A typical resistor is designed to handle one watt or less without burning up. Our power supplies will easily supply over 100 watts of power! As a result, we have to be careful to keep the resistors cooled by keeping them immersed in water. You can easily burn yourself with one of these hot resistors, so please be careful.

We have a number of $5.6\ \Omega$ resistors. How much power is produced in the resistor if 1 V is applied across this resistor?

How much if 2 V is applied across the resistor?

We would like to apply a voltage that produces between 20 and 40 watts. Find a value of voltage that will do this.

We are going to use the resistor as a heater to heat a small amount of water. How do you expect the temperature of the water to vary as a function of time if the power lost in the resistor is a constant? Explain.

Put 100 ml of water in a styrofoam cup. Place a thermometer and a resistor in the cup as well, with the resistor connected to the power supply. Record the temperature every minute with the power supply adjusted to supply the voltage you calculated on the previous page. Make sure to stir the water to keep the temperature fairly uniform in the cup. You also should note the current each time and check to see the voltage stays constant (don't rely on your calculated values, since R changes with temperature), and note any other phenomena you might observe.

Now turn the voltage down on the power supply. How does the heating rate change? Explain how you measured this.

Do your results agree with P = VI? Explain.

With voltage measured in volts and current in amperes, the product is Joules per second, or Watts. With this in mind, can you calculate the approximate value of the heat capacity of water in Joules/(g·^{o}C). You may need to know that 1 ml of water has a mass of about one gram.

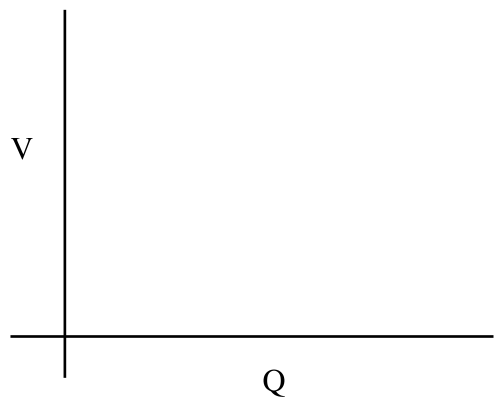
How does this compare with the accepted heat capacity of water of 1.00 cal/(g·^{o}C), where 1 cal = 4.18 Joules?

Finally, let's return to the capacitor, and consider the process of charging it to find the total energy stored in the capacitor. We will redo a calculation you probably did for homework in a rather formal way; here we will use a more graphical approach.

Guidebook Entry VIII.9: Total Energy Stored in a Capacitor

We learned earlier that the amount of charge that one can store in a capacitor was directly proportional to the amount of voltage one used to push that charge onto the plates of the capacitor. The proportionality constant is the capacitance C.

Sketch on the axes below what the voltage looks like as a function of the charge on the capacitor. Label one point along the graph, and explicitly give V at that point as a function of the Q at that point.

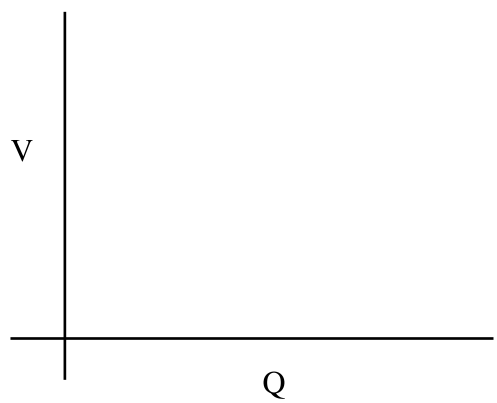


Now imagine that we have the capacitor charged to a voltage V. We then add a tiny bit more charge dq. How much, in terms of dq and V, does this increase the energy of the capacitor? Explain.

Sketch a geometric figure on the graph that has an area corresponding to this change in energy.

Now extend this process another bit dq, and then another bit dq. What area corresponds to these changes in energy? To the total change in energy with the three dq's?

What area corresponds to the total energy stored in the capacitor if it is charged to voltage V? Repeat your sketch from the previous page, and shade in the area corresponding to the stored energy. Using the fact that the area of a triangle is half the base times the height, give a mathematical expression for the stored energy in terms of Q and V.



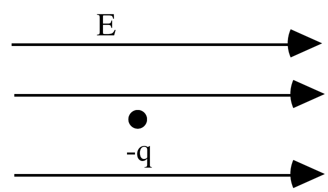
Session 3: Conductors and Applications of the Electric Potential

In this session, we consolidate some of our ideas about the electric potential in combination with our practical experience with electrical conductors.

First, we need to make sure we understand what a conductor is; the first exercise is a set of questions for you to think about that will help sharpen your view of conductors.

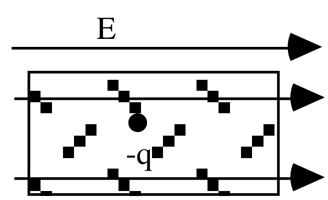
Guidebook Entry VIII.10: A Microscopic View of a Conductor

A small, negatively charged object (say an electron) is placed in an electric field as shown:



What direction is the force on that charge? Sketch a force arrow in the figure. If the charge is initially at rest, what direction does the charge move?

A conductor is a material that conducts electricity; in other words, at least some of the charges that are the constituents of all matter are more or less free to move around in that material.



If we apply an electric field to the conductor, what force does the charge feel? Sketch an arrow of force on the figure.

What direction does the charge move, if it is initially at rest when the electric field is applied?

A real conductor has many charges in it that are free to move. Imagine that the conductor has a uniform distribution of charges that experience the same force. Does this result in an electric current? Explain. If yes, what direction does the current flow?

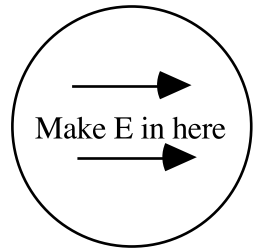
In your own words, describe what happens in a conducting wire on a microscopic scale (in terms of fields, forces, and charges) when you connect that wire from one end of a battery to another.

In this next section, we'll investigate what happens to a conductor which is placed in an electric field, but is *not* connected to a complete electric circuit. This is particularly simple to analyze, and is even practical in a number of circumstances. We will draw on our experience with real conductors to perform what Einstein called a *gedankenexperiment*, a thought experiment--one that only exists in our minds.

Guidebook Entry VIII.11: A Conductor at Equilibrium

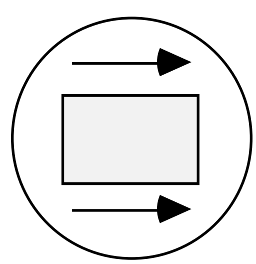
We want to investigate the effect of a uniform electric field on a block of conducting material, say aluminum or copper. But first we have to produce the electric field. We want to produce the field with some electric charges that we imagine we are able to place where ever we like. On the experimental playing field in the figure below, show where you would place charges to get a nearly uniform electric field in the region of the circle, which is where we will then place our conductor.

Put charges out here

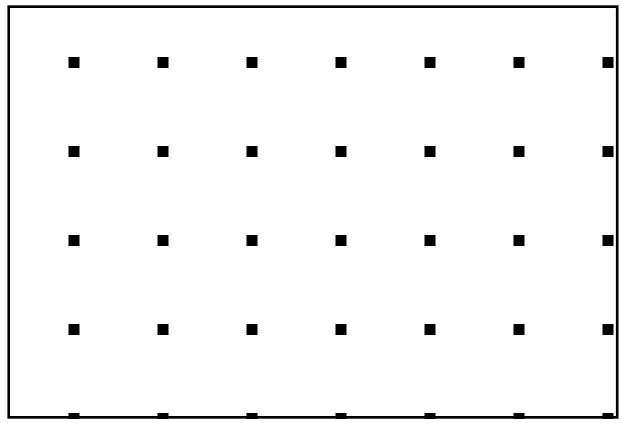


Check your prediction with the computer program E-field. What works better, using points or lines? Why? Explain.

Now, we place a conductor within the experimental circle. Charges in the conductor begin to move. Where do they go? Sketch net charges on the conductor with + and - figures.



Below is a magnified picture of the conductor. Resketch the charges on this new picture of the conductor. Now, consider the electric field produced **not** by the external charges, but just by these charges on the conductor itself. What direction is the electric field produced by these charges?



How does this direction compare with the direction of the original electric field, the one produced by the external charges?

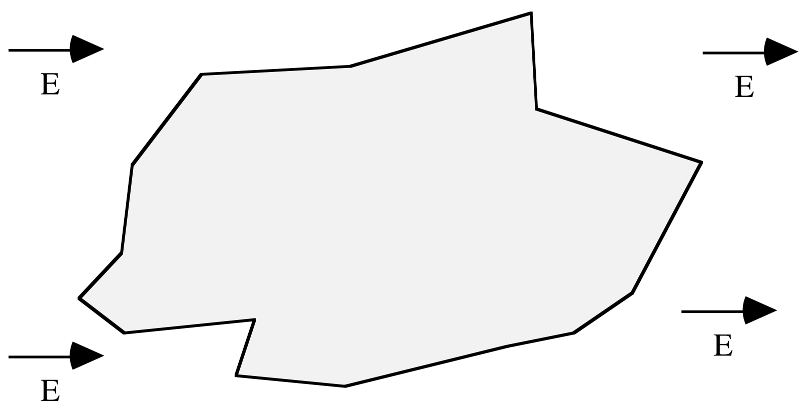
Does this make the *net* electric field inside the conductor get bigger or smaller than the orginal electric field, the one produced by the external charges?

Will charges continue to flow forever? If so, where do they go? If not, what determines when they stop? In particular, what is the final value of the electric field inside the conductor? Explain, and then check with an instructor.

An important property of conductors that you should have discovered above is that, as long as there is no complete circuit (i.e. we can consider this a static situation), is that the electric field inside the conductor is zero. If the electric field was not zero, then the charges would move until they produced internal electric fields that canceled out the external fields. This is also a good approximation for the connecting elements in most electric circuits that contain resistors and capacitors. Let's now apply Gauss's law to see where the charges end up.

Guidebook Entry VIII.12: Where do the Charges Go? Using Gauss

Consider a piece of conductor placed in an electric field, as sketched below.



What is the electric field inside the conductor? Explain.

Now draw in an arbitrary Gaussian surface (cylinder, box, sphere, whatever you like) completely within the conductor. What is the electric flux (recall this is E times the surface area, added or integrated up over the full Gaussian surface)? Explain. Check with an instructor at this point.

What does this mean about the charge within your Gaussian surface?

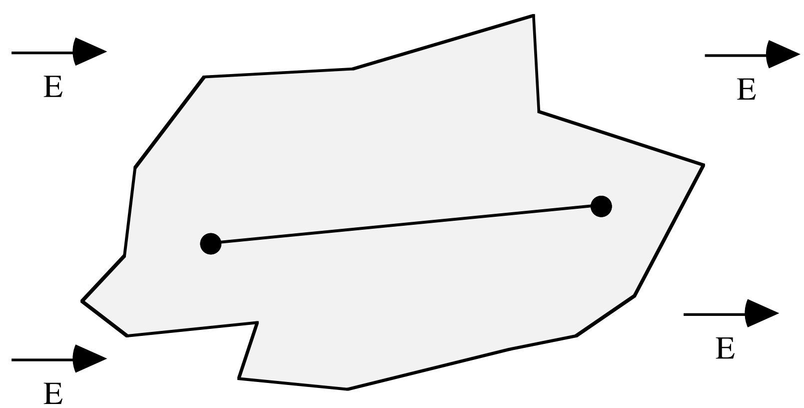
With this in mind, where does the free charge (or more accurately, the charge imbalance) exist in a static conductor?

Does this depend on how good the conductor is? Why or why not? This question is a little subtle, so you might want to discuss it with your instructor.

Now we will see how this can be interpreted in terms of the electric potential in a conductor.

Guidebook Entry VIII.12: Electric Potential in a Conductor

Once again consider a piece of conductor placed in an electric field, as sketched below.



I have marked two points within this conductors (they are just convenient imaginary markers that do not move about; they are not free charges). I have also drawn a line from one point to the other within the conductor. This is also purely conceptual, and is not meant to indicate physical motion or field or force.

The electric field will cause charges initially to move, which will eventually settle down. Where do they end up? Mark them qualitatively?
What is the electric field along the line between the two points?

Recall that the potential difference between two points can be calculated from the electric field by integrating the electric field along a path from the first point to the second. In the case of a uniform field, this integral is just the electric field times the distance. Write these two expressions as equations.

What then is the potential difference between the two points?

What is the potential difference between any two points in the conductor? Why?

Is your result in agreement with the way you used wires (which are conductors) in the early weeks of this course? In other words, what was the voltage difference along a wire?

Can you express a general rule about potentials in conductors?