Group____________________________

Unit 4: Direct Current Circuits

In this unit, you will learn the techniques necessary to calculate the current flow through an arbitrarily complex network of resistors and batteries. In the process, you will have a chance to solidify your conceptual understanding of voltage and current. And while the principles you will encounter are specifically applicable to direct current circuits (i.e. circuits where the current flow through each section is unchanging in time), they will often be applicable to circuits with alternating current flow, and to the short term, or transient, behavior of circuits. In addition, these principles can be extended, if one is careful, to include the behavior of other circuit elements which we will encounter in future units, such as capacitors and inductors.

Session 1: Constant Current and Constant Voltage Supplies

To help you understand the response of a circuit to a voltage, and the relationship between voltage and current, it will be helpful to use our power supplies rather than batteries, since power supplies are more nearly perfect batteries. In the following investigations, you will learn how to use the power supplies in constant current, and constant voltage modes, and see how the bulbs respond to those two modes. You can then compare this to how the batteries worked with the bulbs, to see which is the better approximation for a battery which is lighting a bulb. Finally, we'll discuss the limitations of real batteries (which many of you began to see over the past week).

Recall that we began with an electrical model that concentrated on current, in analogy to the flow rate we used to describe water flow. We used the brightness of the light bulbs as indicators of how much current was flowing. We will rely on that again in these exercises, but you should feel free to use your current meter if you wish at any point to investigate the current flow more quantitatively.

Guidebook Entry IV.1: A Constant Current Supply

Our power supplies have the ability to supply a constant, fixed (by the knob) amount of current. This assumes that there is a reasonable path through which to supply that current; the supplies do stop short of creating a huge spark through the air if there are no wires connected! So, in order to set the power supply up properly for the constant current mode of operation, do the following in sequence:

1. Make sure the power supply is turned off, and no wires are connected to the front panel.
2. Turn the power supply on, and adjust the voltage knob to 7 volts (for safety and to preserve bulbs, this will be the most voltage we will allow the supply to produce).
3. Turn the power supply off again.
4. Connect a wire from the "+" output of the supply to the "-" output of the supply.
5. Turn the current control knob to zero (completely counter-clockwise).

6. Now turn the power supply on. For the BK supplies (tall ones), make sure the "HI-LO" current switch is set to "LO."
7. Turn the current control knob up slowly until the meter indicates about 350 mA of current.

Now the power supply will supply 350 mA into whatever circuit you give it, if it possibly can. You may check this with the current meter (on the 500 mA scale) directly if you wish.

Now take two of the smaller, round bulbs (NOT the taller oblong bulbs, or you will burn them out, as I did). Then investigate the behavior of the light[s] when supplied by the power supply in the following configurations:

Singly

Draw a circuit diagram of the power supply supplying current to a single bulb. You may use a box with 350 mA written in it to symbolize the power supply.

How bright is the bulb?

Parallel

Draw a circuit diagram of the power supply supplying current to two bulbs in parallel.

How bright do you predict the bulbs will be in comparison to the single bulb case?

Connect the circuit and see. How bright are the bulbs in comparison to the single bulb case?

Explain in your own words why this is.

Series

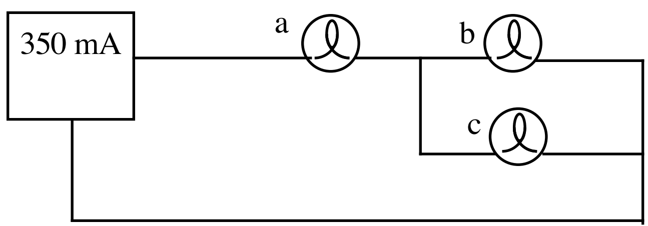
Draw a circuit diagram of the power supply supplying current to two bulbs in series.

How bright do you predict the bulbs will be in comparison to the single bulb case?

Connect the circuit and see. How bright are the bulbs in comparison to the single bulb case?

Explain in your own words why this is.

To see if your really understand what is going on here, we will return to an old puzzle that you wrestled with earlier, adding a bulb in parallel to one of two bulbs in series. In other words, consider two bulbs (a and b) supplied in series by the constant current supply. Then we add bulb c.



Guidebook Entry IV.2: A Puzzle Revisited!

What do you predict will happen to the brightness of all the bulbs when you add bulb c? And how will bulb c compare to the other two?

Build the circuit and record your observations.

Did you see what you expected? If not, can you now explain what you saw?

How did the results compare to what you saw when you did this with a battery? Double check your memory by actually doing this with a battery!

Now we will take up the constant voltage case. In this situation, the power supply does its best maintain a constant potential difference, or voltage, between the "+" and "-" terminals. If it tries to do this across a very good conductor (say a stout copper wire) connected to the two terminals, it will not be able to supply enough current to do so. But across a reasonable circuit (one with some resistance in it), it will do this very well.

Guidebook Entry IV.3: Constant Voltage Supply
In order to set the power supply up properly for the constant voltage mode of operation, do the following in sequence:

1. Make sure all wires are disconnected from the supply.
2. Turn the voltage control knob to zero (completely counter-clockwise).
3. Turn the current control knob all the way up (completely clockwise)
4. Turn the voltage control knob up slowly until the meter indicates about 3 V of potential difference.

Now the power supply will supply 3 Volts across whatever circuit you give it, if it possibly can. You may check this with the volt meter, if you wish.

Now take two bulbs (if you are using one of the HP power supplies, you must use two of the taller, oblong bulbs). Then investigate the behavior of the light[s] when supplied by the power supply in the following configurations:

Singly

Draw a circuit diagram of the power supply supplying current to a single bulb. You may use a box with 3 V written in it to symbolize the power supply.

How bright is the bulb?

Parallel

Draw a circuit diagram of the power supply supplying current to two bulbs in parallel.

How bright do you predict the bulbs will be in comparison to the single bulb case?

Connect the circuit and see. How bright are the bulbs in comparison to the single bulb case?

Explain in your own words why this is.

Series

Draw a circuit diagram of the power supply supplying current to two bulbs in series.

How bright do you predict the bulbs will be in comparison to the single bulb case?

Connect the circuit and see. How bright are the bulbs in comparison to the single bulb case?

Explain in your own words why this is.

Finally, let's consider a real battery. How does the battery behave in a circuit?

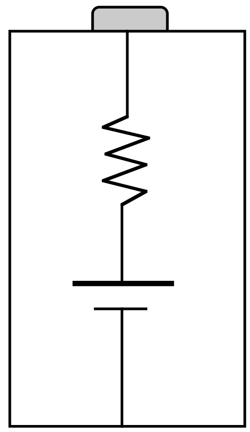
Guidebook Entry IV.4: Real Batteries and Power Supplies

Which of the two cases better approximates the behavior of a battery with our bulbs: constant voltage or constant current? Give an example?

In real circuits, nothing comes out ideally. If you light a single round bulb with 3 V from the battery, and then add a second bulb in parallel, the brightness of the first bulb should not dim. Does it in reality, however? Check with an instructor here.

We can see why this might happen if we approximate a battery as a constant voltage source in series with a resistor (you may consider it as another bulb, if you wish). Draw a diagram of such a configuration, and use it to explain what happens when you try to draw more current out of the supply than the supply can comfortably give.

To a reasonable approximation, real batteries look like constant voltage supplies with a series resistor, called the internal resistance.

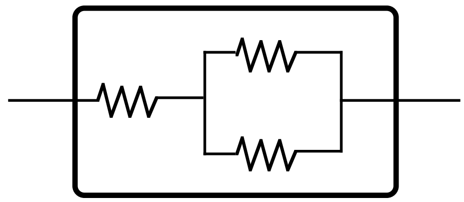


Schematic view of
a real battery

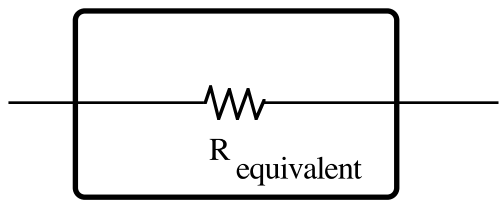
Real power supplies also have their limitations, but are a bit more complicated to describe, and the exact details of those limitations vary from one supply to another.

Session 2: Series, Parallel, and Network Circuits

In this session, you will apply the basic rule of Ohm's law, V=IR, to circuits containing several resistors. For many of these circuits, you can use the fact that they will behave like an equivalent, single resistor. In other words, if we were to place a combination of resistors into a box, such as the one shown below



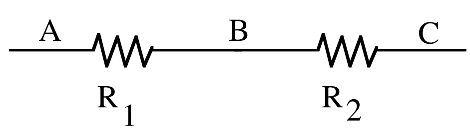
it would be impossible to tell this box (from the electrical properties as measured from the two outside leads) from a similar box containing a single resistor, if we choose that single resistor to have the correct value, which we call the equivalent resistance.



More important than being able to find equivalent resistances (which after all, we could always measure with an ohmmeter anyway), dealing with problems such as this will help solidify our concepts of current, voltage, and resistance.

Guidebook Entry IV.5: Series Resistors

Let's review the case of two series resistors. This should be review, but it is so important that we want to make sure everyone understands the concepts.



If some current is flowing through R_1, say 20 mA, how much current is flowing through R_2? Explain.

If the voltage drop from A to B (across R_1) is 4 V, and the voltage drop from B to C (across R_2) is 3 V, what is that drop from A to C (across both resistors)? Explain.

Use Ohm's law to calculate the value of the R_1.

Use Ohm's law to calculate the value of R_2.

The equivalent resistance is that resistance that will obey Ohm's law for the combination, where V is the total voltage change across the pair of resistors. What is the equivalent resistance for this series combination? Make sure to verify this using the current and voltage difference you calculated above. Check your reasoning with an instructor.

Imagine that we now have arbitrary values of R_1 and R_2. Allow the current through the pair to be I. What is the voltage difference across R_1?

What is the voltage difference across R_2?

What is the total voltage difference across the pair?

What then is the equivalent resistance, that is, V_{total} divided by the current?

Verify this rule by taking two resistors, measuring their resistances with your ohmmeter:

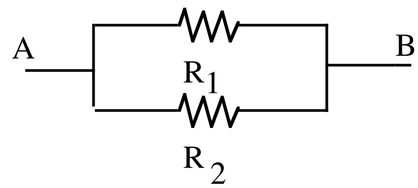
and measuring their resistance in series:

Does this agree with your rule?

You should now have verified an important rule that we have already used: series resistances add. In the next activity, we will develop a similar rule for two resistances in parallel. This rule is less obvious, but no less useful.

Guidebook Entry IV.6: Parallel Resistors

Let's now consider the case of two parallel resistors.



If some total current is flowing in through connection A, say 50 mA, and some through R_1, say 20 mA, how much current is flowing through R_2? Explain.

If the voltage drop across R_1 is 4 V, what is the voltage drop across R_2? Explain.

Use Ohm's law to calculate the value of the R_1.

Use Ohm's law to calculate the value of R_2.

The equivalent resistance is that resistance that will obey Ohm's law for the combination, where V is the total voltage difference across the pair of resistors. What is the equivalent resistance for this series combination? Make sure to verify this using the current and voltage you calculated above. Check your reasoning with an instructor.

Imagine that we now have arbitrary values of R_1 and R_2. Allow the voltage difference across the pair to be V. What is the current through R_1?

What is the current through R_2?

What is the total current through the pair, that is the current flowing in at A or out of B?

What then is the equivalent resistance, that is, V_{total} divided by the current?

Verify this rule by taking two resistors, measuring their resistances with your ohmmeter:

and measuring their resistance in parallel:

Does this agree with your rule?

The rules for adding resistors in series and parallel are most often summarized as follows:

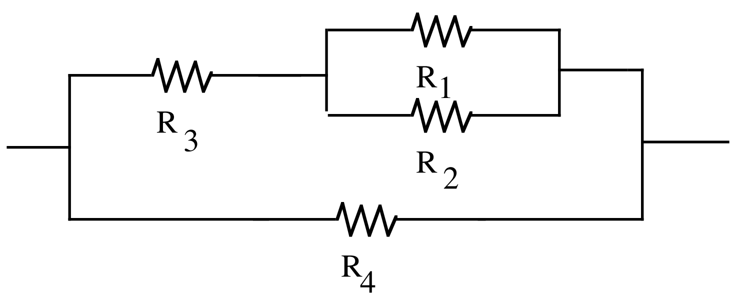
$$Series: \quad R_{total} = R_1 + R_2$$

$$Parallel: \quad \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Now that we have these rules, we can apply them to more complicated systems. We'll consider such an example in the next activity.

Guidebook Entry IV.7: A More Complicated Network of Resistors

Let's try to find an equivalent resistor for the following network of resistors.



Let's also imagine that we also know the values of these resistors:

$$R_1 = R_2 = R_4 = 2\Omega$$

$$R_3 = 1\Omega$$

We typically start at the "innermost" set of resistors, in this case R_1 and R_2. What is the equivalent resistance of these two, which we will call R_{12}?

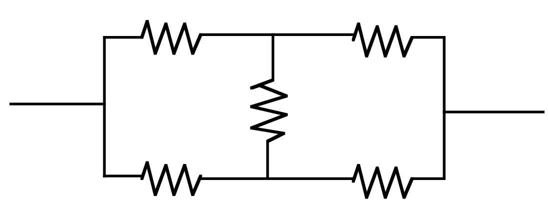
Now draw a new schematic that replaces R_1 and R_2 with a single resistor labeled R_{12}.

What is the equivalent resistance presented by the combination of R_{12} and R_3, which we will now call R_{123}?

Redraw the schematic again, replacing R_{12} and R_3 with a single resistor labeled R_{123}.

Finally, what is the total equivalent resistance of the full network?

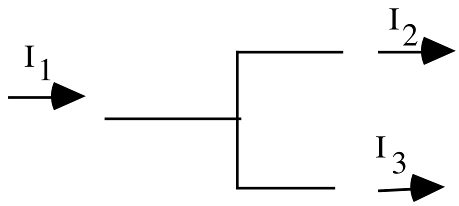
This technique of successive replacement of series and parallel combinations of resistors is quite powerful, but there are some circuits that simply cannot be reduced by this technique. Some of them have resistors that defy classification as either series or parallel, as in the following case:



One can also be frustrated by having more than one battery in the system, where the battery is actually within the network of resistors, as in the next activity. We will use a pair of rules known as Kirchhoff's rules to allow us to solve for the current flowing through each circuit element by generating a set of equations that we can solve simultaneously.

Guidebook Entry IV.8: Use of Kirchhoff's Rules

You have already become familiar with Kirchhoff's rules, although we haven't refered to them by this name yet. The first that we will consider has to do with what happens at junctions. At junctions (i.e. where at least three paths join), the current values usually change in going from one leg to another. They do not do so capriciously, however. For example consider the junction made by the joining of three wires.

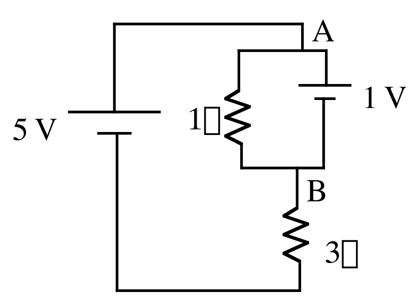


What is the relationship between I_1, I_2, and I_3? Express this relationship as a mathematical equation.

This first of Kirchhoff's rules is often expressed as the total current into a junction equals the total current out. Alternatively, some people prefer to call a current *into* a junction positive and a current *out* of a junction negative, so the junction rule, taking into account the sign of the currents, becomes "the sum of the currents into a junction is zero."

The second of Kirchhoff's rules uses the fact that the voltage has a particular value at any point, just like pressure in a stable current flow. As a result, one can find the voltage/pressure difference between any two points by counting up the voltage/pressure drops between those two points. If there are two possible ways of getting from the first point to the second point, each path must give the same voltage/pressure difference.

Let's be more concrete. Consider the circuit below.



The voltage difference between point A and point B must not depend on the route traveled. What is this voltage difference if one travels through the 1 V battery?

What is the voltage difference if one travels through the 1Ω resistor? Express your answer in terms of the current flowing through that resistor, as well as in volts.

What must be the sum of voltage changes if we travel in a complete loop?

The most common way of expressing the second of Kirchhoff's rules is that the sum of voltage changes over any loop is equal to zero. The individual voltage changes depend, of course, on the direction in which we are traveling. Moving from the negative end (shorter line) to the positive end (longer line) of the battery increases the value of the voltage by the voltage of the battery. Moving in the direction of current flow through a resistor decreases the value of the voltage by $V = IR$. Moving in the opposite direction for either of these cases, the sign of the voltage change is naturally opposite.

Before we can do add up these IR terms, we need to define the currents with a symbol, including what we expect the current directions to be. Since the current must be the same all along a wire until we reach a junction, we define a unique current for each length of wire, running from junction to junction. It is not important that we guess the direction of current flow correctly, but rather just that we have a consistent convention. For this purpose, assume that I_1 travels up through the 5 V battery, I_2 travels down through the 1Ω resistor, and I_3 travels up through the 1 V battery. Label the sketch on the earlier page!!

Given these rules, you should be able to write a current conservation equation for each of the junctions. Do so here:

Does it make sense that the two equations are the same? Explain.

Now, write three equations for our circuit using the three possible loops. Check your answer with your instructor.

In general you will get more equations than you need. Here we are searching for three currents, and therefore need three equations. You can show in any individual case that the excess equations are redundant; they can be obtained from the other equations.

Now all that remains to solve this problem is to solve the equations, which you will do for homework.

Session 3: Practice with Kirchhoff

Before we start, let's review the Kirchhoff's Rules process:

1) Define a separate current for each circuit leg, i.e. from each junction to the next junction by each possible path. Label your currents, and include a defined direction (which is arbitrary).
2) Write junction equations for each independent junction (typically all but one). Make sure to pay attention to your *defined* current direction.
3) Write loop equations for each independent loop (typically all but one). Here you must carefully assign signs to each voltage change as you travel an imaginary trip around the complete loop:
a) negative to positive on battery is a positive voltage change, positive to negative on battery is a negative voltage change,
b) moving with the defined current direction across a resistor is a negative voltage change, moving against the current direction is a positive change.
4) Solve the resulting equations simultaneously for (typically) each current. You must have an independent equation for each unknown. For problems that warrant this technique, you will have to have at least one junction equation, and two loop equations.

Application of Kirchhoff's rules is a complicated business, as you now know. To gain some confidence in the process involves practice. Since Kirchhoff is not needed for simple circuits where series and parallel resistors can be reduced to equivalent resistances, it is usually only practiced on complicated networks that are very difficult to understand. However, to help you feel more comfortable with Kirchhoff, in this session, you will practice using Kirchhoff's rules with familiar circuits that you can solve with series and parallel equivalent resistances. With luck and persistence, this will give you some insights that will help you with the more complicated problems. So, in case it seems like you are solving easy problems repeatedly using complicated tools, you are exactly right! Be prepared for this!

Guidebook Entry IV.9: Series Resistors

a) Consider a circuit that consists of a source of voltage difference (often, in the case of batteries and power supplies, called potential difference) V (a battery) and two resistors (R_1 and R_2) in series. Draw a circuit diagram for such a circuit, labeling the elements.

b) From your knowledge of series resistor circuits, what is the current that flows out of the battery? Explain briefly.

c) Now use Kirchhoff's loop rule to derive an equation that relates the voltage difference, resistances, and current flow. To do this, we first must define a direction for current flow. In your original circuit schematic above, define and draw a current flow I in the direction that the current actually flows (i.e. out of the positive end of the battery).

d) Now we have to consider a complete trip through the circuit, as if we were a bit of charge traveling through the circuit. As we make one complete trip through a closed loop, we experience a series of voltage changes that all add up (algebraically--i.e. including the "+" and "-" signs) to no net voltage change. Going across a battery from "-" to "+" is an increase in voltage, and going in the direction of current flow across a resistor is a decrease in voltage of magnitude IR, which typically enters as a –IR in your equation. **Starting at the negative side of the battery and going first through the battery,** make such a trip, and write the resulting equation. Solve this for the current flow, and show that it is the same as what you got by equivalent resistors. Make sure everyone in your group gets the same answer. If you don't get this easily, talk to an instructor!

You now have solved the problem, but next you will redo it to examine the effects of sign and direction conventions on your solution.

e) Now repeat the exercise, only this time start in between the two resistors. Travel in the same direction as you did before. What do you discover?

f) What do you expect to happen if you start at the **positive** side of the battery for your trip, and travel through the battery first (i.e. in a direction opposite to your previous trip)? How do you expect this will change the resulting equation?

g) Now, explicitly do this calculation. How does your result differ from the equation you got in part d)?

h) Redraw your original circuit. Now define your current flow to be in the opposite direction from what you defined in part c). Use the same trip direction you used originally in part d) (start at the negative side of the battery, go through the battery first). Solve for the current. Compare this to your previous result, and explain. Make sure to check with an instructor at this point.

i) Summarize in your own words what happens to the resulting equation when you reverse the direction of your imagined trip around the circuit loop. Compare your answer with your partners.

j) Summarize in your own words what happens to the resulting equation when you reverse the direction of your current definition. Compare your answer with your partners, and check with an instructor.

k) Imagine that $V = 1.5$ V, $R_1 = 100\ \Omega$ and $I = 5$ mA. What is the value R_2?

The series resistor case is so simple that it doesn't even have anyplace to apply the junction rule. The simplest circuit in which we can apply the junction rule is a battery with two parallel resistors, which we investigate in the following section.

Guidebook Entry IV.10: Parallel Resistors

a) Consider a circuit that consists of a source of potential difference V (a battery) and two resistors (R_1 and R_2) in parallel. Draw a circuit diagram for such a circuit, labeling the elements.

b) From your knowledge of parallel resistor circuits, calculate the current that flows out of the battery. Explain briefly.

c) What is the voltage difference across R_1? What current flows through this resistor?

d) What is the voltage difference across R_2? What current flows through this resistor?

e) How is the total current related to the current through R_1 and the current through R_2? Express this total current in terms of our results in c) and d). Does this agree with your result in b)? Check your results with your partners and an instructor.

f) Redraw your circuit diagram, and define current flow in each section. How many different currents do you need to define? Assign them each the direction that you know current must flow.

g) There are two junctions. Apply Kirchhoff's junction rule at each one, and write the resulting equation. Compare them to each other, and to the result in e). Explain your results in your own words.

h) Use the Kirchhoff loop rule on each one of the three possible [simple] loops to derive three different equations. Be explicit about the direction of travel you use for your calculation by showing them on another sketch of the circuit. Make sure to be careful with signs!

i) Take any two of the equations you got in part h) and combine them algebraically to get the third. Check with your partners and an instructor.

j) Solve an appropriate set of equations to get the current through the battery. Compare your result to the result you got in part b).

You have now solved the problem for the third time, this time using Kirchhoff , but just as we did for the series case, we'll now investigate how the Kirchhoff equations are affected by our current and loop definitions.

k) Now change the *defined* direction of current flow through R_2. Reapply the junction rule. How does the resulting equation change?

l) Now use the loop rule on the loop that includes the two resistors, using the new definition of current flow from part k). How does this compare to your previous equation for that loop?

m) Use this new definition of current flow from part k) and derive enough equations to solve for the current flow through R_2. Solve for the current through R_2.

n) Summarize in your own words what happens to the terms in the various equations when you change the defined direction of a current.

o) Summarize in your own words what happens to the terms in the various equations when you change the direction of your imagined trip through a loop.

p) If the values of the resistors are $R_1 = 100\ \Omega$ and $R_2 = 200\ \Omega$ and the value of the current through the battery is 45 mA, what is the voltage of the battery.

One major purpose of this session has been to make clear what the effect of the various sign conventions are on the equations that are developed, but that the final results should always be independent of our initial sign conventions. In other words, the actual current through a resistor (direction and magnitude) doesn't depend on how we initially guess the direction of current flow is, or what direction we choose for our imagined loop trips that we use for creating loop equations. This would indeed be a surprising interaction of our minds with reality if we could change the direction of electron motion by how we think about it!

One other thing you should have noticed—we get more equations than we need to solve the problem. Which ones should be used? We should have as many equations as we have unknown currents. If we have too few, we can't possibly get a solution. If we have too many, we can get two equations to combine to give us the enlightening equation 0 = 0. We should have each resistor appear in at least one loop equation, and make sure that each current appears in at least one junction equation. For simple networks, this is easy, but for complicated networks, it can be quite tedious and confusing. Persistence and careful attention to sign conventions are the keys to success!