Unit 2: Circulating Fluids

In this unit, we will investigate some principles that apply to closed circuits of fluids. These systems have some practical applications (e.g. to fluid based heating and cooling systems, to refrigerations systems, and to circulatory systems in animals), but also illustrate some concepts that we will later find useful in understanding some seemingly very different systems, such as electric and magnetic fields.

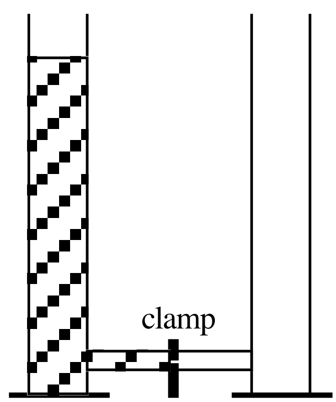
Session 1: Approach to Equilibrium

In this session, you will wrestle with some of the same issues you saw when you watched how a cylinder emptied through a small capillary, only now you will deal with some of the details more carefully. You will use two identical cylinders with a flexible tube connecting them. Instead of using a glass capillary to restrict the flow between the two cylinders, you will use an adjustable clamp to allow you to vary the restriction.

The first exercise should be rather easy, but it is essential that you understand the ultimate fate of our two cylinder system.

Guidebook Entry II.1: Steady State, or Where We End Up.

You will start out with two cylinders connected together as shown. Make sure that the two cylinders have blank-off (i.e. closed) tubes connected to the outside barb connectors. Furthermore, make sure that there is a clamp firmly closing the flexible tube between the two cylinders. Then fill either one of the two cylinders. Check to make sure there is no flow through the connecting tube; if there is some flow, tighten the clamp until it stops.



Soon, you will open the clamp between the two cylinders. Before you do, predict where the final level of the water will be in each of the cylinders.

After you have written your prediction down, discuss this with your group. When you have agreed, make a mark on each of the cylinders where you expect the water levels will end up. Now, open the clamp, and let the water flow. How close were your predictions? Explain any discrepancies.

Imagine now that the left tube is filled with water and the right tube filled with oil to an equal height. What, if anything, do you think would happen when you open the clamp between the two? Discuss this with your partners, and write your prediction down.

Now, go look at the demonstration tube on the lab bench. Does this agree with your predictions? Explain what you see.

For the draining of a single cylinder, we found that the time dependence was roughly a decreasing exponential. We want to find out if that is true for our two cylinder system, and more importantly, how we can change the characteristic time of the system. To do this, however, we first have to understand the problems associated with defining the characteristic time.

Guidebook Entry II.2: Draining Time--Defining the Characteristic Time.

In this exercise, we want to develop a quantitative way of describing how quickly a process proceeds when it is governed by a decreasing exponential law. First, you have to understand the difficulty of defining the draining time for flow that has an exponential dependence. To help with this, there are a couple of Excel spreadsheets that show the difference between a constant draining rate (linear) and a draining rate that is proportional to the height (as we saw last week).

Open the Linear Draining worksheet from the Workshop Data folder. If this is a graph of the height of water in one of the cylinders as a function of time, what is the amount of time it takes to drain the cylinder? Explain.

Now open the Exponential Draining worksheet from the Workshop folder. If this is a graph of the height of water in one of the cylinders as a function of time, what is the amount of time it takes to drain the cylinder? Explain. Make sure to discuss your answers within your group, and talk to an instructor about your conclusions.

Print a big copy of the exponential graph with scale lines drawn in. Find the time it takes for the graph to fall to 1/e (where e=2.718) of its original value. Does this time depend on what you call the original value? For example, choose a time roughly halfway into the graph and call this height the "original" value. Is the 1/e time the same as or different than before? Explain. Make sure to discuss your answers within your group, and talk to an instructor about your conclusions.

The time it takes for an exponential function of time to decreas by a factor of 1/e is called the characteristic time. You should have discovered that, while it takes an infinite amount of time to complete the process, the characteristic time is well defined, and easy to measure from the data. One sometimes also defines the halving-time, or half-life, which is the time required for the height (or whatever the relevant quantity might be) to drop to half of its original value. The relationship between the two is:

$$T_{1/2} = (\ln 2)\, T_{1/e}\,.$$

This relationship should be easy to verify for any function $f(t) = Ae^{-bt}$ by examining the times $T_{1/2}$ and $T_{1/e}$ as defined implicitly by the relations $f(T_{1/2}) = A/2$ and $f(T_{1/e}) = A/e$.

Guidebook Entry II.3: Tube Draining--Height as a Function of Time

First, you should verify that the draining rate for one cylinder into another looks qualitatively exponential. Do as you did for draining a single cylinder, except now use the adjustable hose clamp as the flow restriction. Before you actually do the measurement, adjust the clamp to give you a reasonable flow rate; one that you can easily measure with a stop watch, but that does not take an unpleasantly long period of time. Then apply a second clamp to the tube to close it off completely, so that you can fill only one cylinder, and have a well defined starting time. Then take data as before. You will want to make marks on the side of the tube, or tape a ruler to the side of the tube. You may also want to make use of the "split time" feature of the stop watch to read off intermediate times as the watch continues to time (this will depend on whether you want equal time measurements, or equal height measurements).

height	time

Now graph this as an x-y graph. Does this look like a decreasing exponential? In what way is it different? What kind of function looks like this data? Make sure to discuss this with your partners.

In a homework problem, you were asked to find an equation for height as a function of time for a draining tube that included the constants that related pressure to flow, and pressure to height. You should have found that the height behavior as a function of time looked like

$$h = h_o\, e^{-t/\tau}$$

where h_o is the initial height, and τ is the characteristic time. You did not solve for this precise form, but rather had instead of τ a product or quotient of two of the proportionality constants. Now we will take the opportunity to see how τ depends on some of those proportionality constants.

Guidebook Entry II.5: Relaxation Time--Understanding the Dependence on Cylinder and Restriction.

Now you will be able to simply measure a single time that is indicative of the time dependence of the system. You may choose that to be the half time (the time it takes the fluid level to reach halfway to the equilibrium level), or the characteristic time (the time it takes the fluid level to reach to within 1/e of the equilibrium level). Either of these is fine; just be explicit which you are using.

First, what to do predict the characteristic time will do as you open the the restriction in the connecting tube?

Now measure the characteristic time for several different openings. You may wish to measure how open the clamp is by how many turns it is open from the completely closed position. Does it do what you expected? How does it seem to depends on how open the clamp is?

Now, we are going to change the constant that relates the height not to the pressure, but to the volume. To do this, we will stick a wooden dowel in each of the cylinders to change the height-to-volume relationship. How do you predict this will change the characteristic time?

Now make the measurement? Does it do what you expected?

Session 2: Pumps and Pressure Drop Over a Circuit

In this session we will investigate a closed circuit system with a pump. Once we introduce a pump, we no longer are restricted to systems that are static, or have a transient (short-lived) flow, but a system that has continuous flow.

For our pump, we will use a common and inexpensive hand operated pump, otherwise known as a watergun. This pump has the advantage of being very inexpensive, and requires no instruction manual, however it has the disadvantage that they wear out easily (so please be gentle), they leak a little bit (don't hold your apparatus over your computer), and they have a tendency to allow water to flow forward through the gun when under pressure that way, but not backwards. This can have some effects on your measurements, especially for slow flow rates, so be forewarned, and don't hesitate to ask an instructor if you are unsure how to interpret this!

Guidebook Entry II.6: A Constant Flow Source.

If you operate the pump at a constant stroke rate, the flow is roughly constant. With your partners, design a simple experiment to test this. Make sure that you consider how to maintain the pump rate at a nearly constant tempo. Describe your experiment here.

Now perform your experiment, and describe your results.

How do you expect the fluid flow to depend on the rate at which drive the pump? For example, how do you think the fluid flow would behave if you pump at twice the tempo, or at half the tempo?

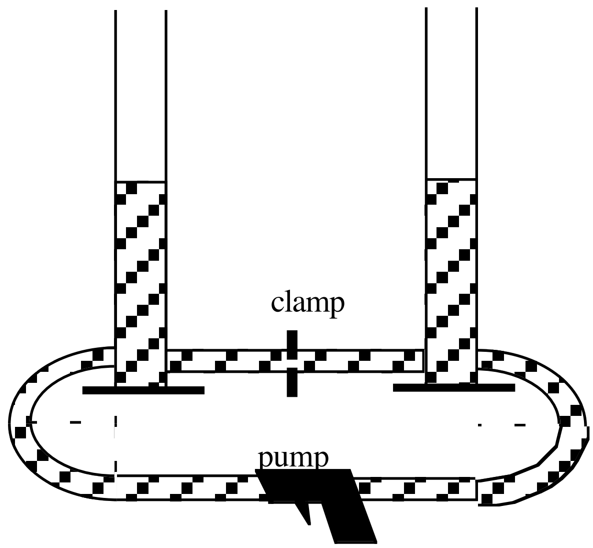
Test your prediction with an experiment. What are your results?

From here on, let's assume that we can approximate our pump as a source of constant fluid flow for a given pump rate. Now with this assumption, we'll see how we can use pressure differences as an indicator of fluid flow rate.

Guidebook Entry II.7: Pressure Difference as a Flow Indicator

We have already seen that we need a pressure difference to get fluid to flow through a narrow restriction. Now we should be able to turn this around and use pressure differences to indicate that fluid is flowing.

With your pump between two cylinders, and the two cylinders also connected with a short tube, you can make a complete fluid circuit. Use a tube clamp to cause a restriction in the short tube, such that there is a significant reduction of flow between the two cylinders. Fill the system such that the cylinders are about half full of water.



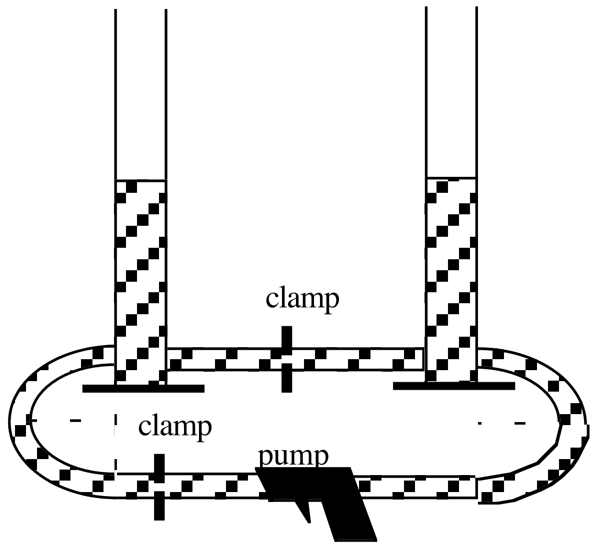
What do you expect to happen if you start pumping at a slow rate?

Do this, and record your observations.

Now, what do you expect to see if you pump at twice your previous rate?

Try this, and record the results. (Also mention how you determined you were pumping at twice the rate.)

Now place a second flow restriction in one of the tubes connected to the pump (i.e. between a cylinder and the pump, *not* between the two cylinders). What do you expect to see if you pump at your original, slow rate?



Do this experiment. Record what you see. Discuss this with your partners, and an instructor. This is a particularly subtle interpretation; make sure to talk to your instructor!

We have been implicitly making the assumption that the only significant source of viscous drag in our system comes from the flow restrictions that we introduce with the tube clamps. In the next exercise we will investigate the drag that is inherent in the tubes themselves.

Guidebook Entry II.8: Flow Through Non-Ideal Tubes.

You need to have three lengths of the thinnest tubing: one about 50 cm, one about 1 m, and one about 2 m. Place the shortest one in the circuit as the connection between the two cylinders. Fill the cylinders about half full of water. Remove all restricting clamps. Pump at a comfortable rate (say one pump per second).

Now, record the pressure drop across the tube. Describe how you took your data.

How do you expect the pressure drop to change if you replace the short tube with the medium length tubing? Why? Discuss your prediction with your partners.

Repeat your measurements for each of the remaining lengths of tubing. Graph pressure versus tubing length, and sketch below.

We can define a new quantity which we will call the flow resistance (R) as the ratio of the pressure difference (P) to the amount of fluid flow (f), or in equation form

$$R = P/f.$$

So, our capillary that we used in the last unit would have a very large flow resistance (very little fluid flows through it), and a big tube has a very small flow resistance (easy to get much fluid through it, even with small P).

What resistance do you predict will result from two fluid circuit elements that are put one after another (or in series, as the jargon goes)? Does this agree with your results with the longer tubes? Discuss with your partners, and explain.

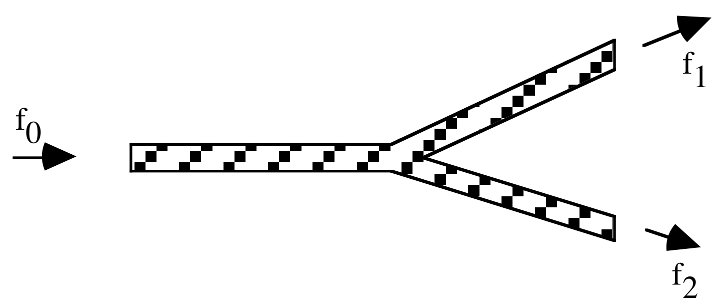
Session 3: Flow in Two Dimensions

In this session, we will consider the flow of water through systems in which the velocity changes, and be a bit more quantitative about how that velocity changes. All of this will depend on the notion of conservation of fluid. In other words, if the flow through a system is some value f (in volume per time), then the flow at a later point in the system must be the same value. (We have made the assumption that we are neither compressing nor expanding the fluid, which is a good assumption for most liquids at pressures we conventionally deal with.)

Guidebook Entry II.9: Changing Fluid Velocities in Tubes.

There are two ways in which an incompressible fluid flowing through tubes can change its velocity along the path: either the tubing branches, or the diameter of the tubing changes. In this exercise, you will consider theoretically what must happen in each of these cases. Make sure to discuss your answers within your group!

First, imagine that the fluid is initially flowing through a single tube that then branches into two identical tubes.



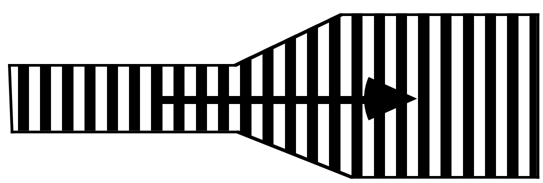
If the initial flow (f_o) rate is some value, say 5 liters/minute, what is the flow rate (f_1 or f_2) through either one of the branch tubes?

What if the initial flow rate is an arbitrary flow rate f_o, what are f_1 and f_2? Explain.

If the initial fluid velocity before the branch is v_o, what is the velocity (v_1 or v_2) in either one of the branch tubes? Again, explain.

Now imagine that the two branch tubes are not identical. Can you write an equation that relates f_o, f_1 and f_2?

Now let's consider the second case in which the cross-sectional area of the tube changes. Let's imagine that the cross-sectional area is initially A_o and then doubles to $A_1 = 2A_o$. What happens to the velocity? How does this compare to the branching example. Explain any differences or similarities.



Finally, if the *radius* of the tube is initially r_o and then changes to r_1, what happens to the velocity? Can you write an equation that for v_1 in terms of v_o, r_o and r_1? Check your result with your partners and an instructor.

Now we are going to consider some more complicated geometry, and do an experimental measurement. We have a tubing connection to the center of two closely-spaced flat acrylic plates. You will introduce water into this center connection, and then observe the flow outward. This is the first situation in which we have *not* constrained the flow to be essentially one dimensional. Here the fluid is free to flow out in a flat plane. Our goal in analyzing this system is to learn how to apply the fluid conservation principle in a more general geometry, and to also develop some familiarity with the velocity function. This is an example of what mathematicians call a vector field, which is a very important concept for the study of electricity and magnetism, as well as fluids.

Guidebook Entry II.10: Fluid Flow in Two Dimensions (or Too Much Mustard on the Burger!).

The initial setup of the two-dimensional flow apparatus is a bit tricky, so you will probably need help. We use a beaker of water on the lab bench as a source of fluid, which we siphon out with a tube. A variable hose clamp is necessary to throttle the fluid flow back to a manageable rate. Once the tube is full, it can be connected to the apparatus (which should be placed in a photo tray to catch the overflow). Place some markings on the top surface as measurement of the distance out from the center. The data you will take will consist of the times at which a dye marker in the fluid passes by each one of the distance markings.

Once you are all set up, you can open the clamp a bit so that there is a steady but slow dripping of fluid from the edges of the plates. You may need to readjust the apparatus to make it more level so that the flow is not just to one side.

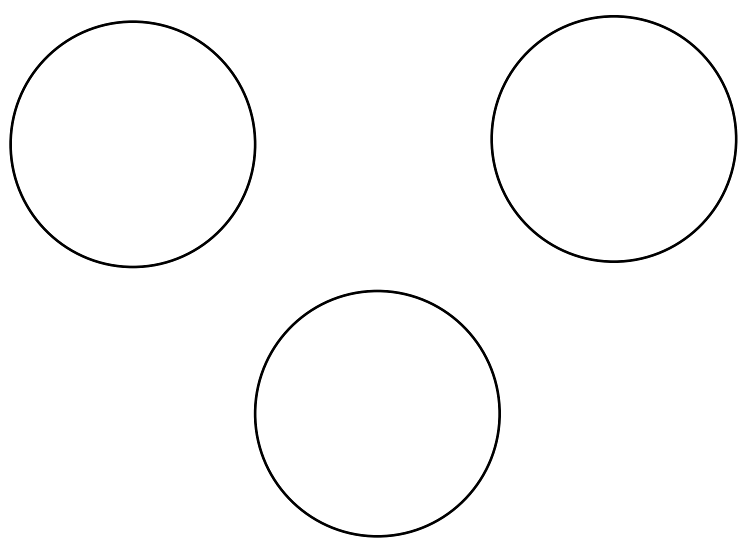
To introduce the dye marker, we use a syringe filled with blue food coloring. We stick the needle of the syringe into the tubing just above the apparatus. A tiny push on the syringe piston will introduce a dye marker that you can easily track through the plates. You should practice this a couple times, and then take some data!

Radial Distance	Time

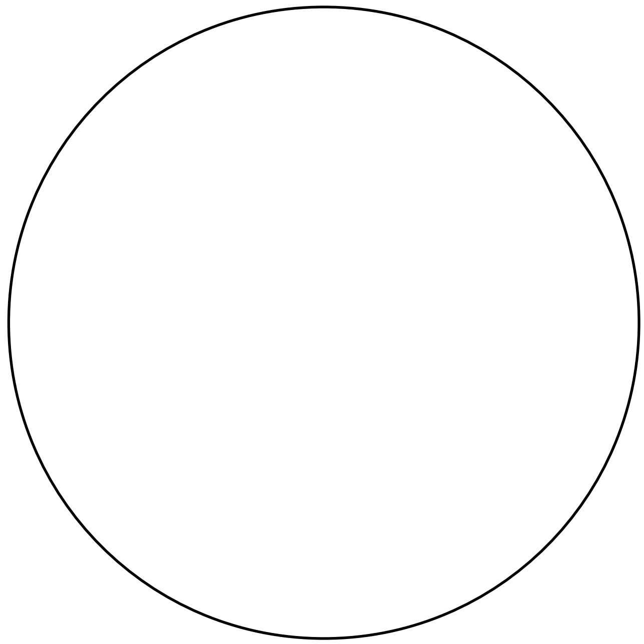
Plot the distance versus time data using Excel, and attach your graph. How is this graph different than what one expects for one-dimensional flow, that is flow restricted to simple tubes with constant cross-section? Explain.

Now make a graph of fluid velocity rate as a function of radius. To do this, you will need to divide a distance (the distance between adjacent markings) by a time (the time difference between those adjacent measurements). This is actually an average velocity. What distance is the most reasonable one to associate with this velocity? Explain. Then do your graph and attach.

You made a measurement of velocity along a particular radius of the disk. Clearly fluid is flowing at *all* points in the disk. Make several sketches of what the dye marker looked like at several times on the circles below.



At any point on the disks, the fluid has a velocity. We represent that velocity as a vector: it has a magnitude and a direction. We can record that velocity in terms of x and y components, or we can represent it graphically by arrows at each point on the disk. The arrows length is indicative of its magnitude (the speed of the fluid flow, regardless of direction) and the direction of the arrow indicates the direction of the fluid flow.



Discuss your sketch with your partners and an instructor. Can you explain qualitatively why the velocity decreases as you get toward the outer edges?

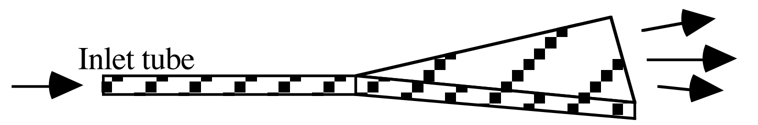
You have now seen *real* flow in a two dimensional system. Now we will consider theoretically what we would expect to see in an ideal two dimensional system.

To do this, we will need to recall the continuity equation which expresses the conservation of fluid through the system.

$$v_1 A_1 = v_2 A_2 \ .$$

Guidebook Entry II.11: Using Conservation of Fluid for Two-Dimensional Flow.

The continuity equation was fairly easy to use when the cross-sectional area was easy to define. For example, in a tube it was just the circular area that we see at the end of the tube. In our disk case, however, the notion of a cross-sectional area becomes rather obscure. Our dye marker can come to our conceptual rescue here though. Before we consider the disk system, let's imagine that the fluid is flowing through a pie-shaped wedge. In other words, the fluid flows from a simple tube into a region defined by some small angle, say 10˚, and a constant spacing t between top and bottom surfaces, as shown.

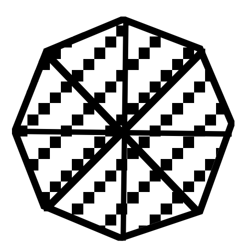


Imagine that we introduced a dye marker into the flow of this fluid. What shape is the front surface of the dye when it is in the inlet tube?

What shape is the front surface once it has entered the wedge shaped region? How does this shape change as the front surface moves along?

Let r denote the distance the dye surface has moved in the wedge region measured from the entrance (the apex of that triangular region). What is the area of the front surface in terms of r and the spacing between the top and bottom surfaces, t?

Now let's consider the parallel plates. You can consider the flow region to be a whole series of wedge regions put next to one another, as shown. If you make the number of wedges greater,



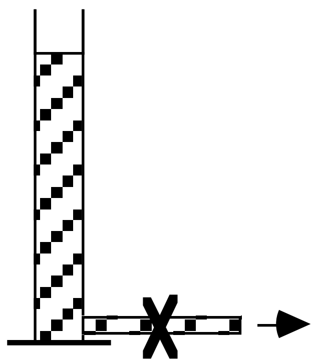
and the angle correspondingly smaller, the figure becomes more nearly a circle. The fluid flows into the center, and out through each one of the wedges simultaneously. What then is the shape of the surface of the dye marker as it is flowing between the two plates (Don't forget that there is some thickness between the two plates.) Make sure you agree with your partners on this one, and check your answer with your instructor.

What is the area of this surface at a radius r from the center?

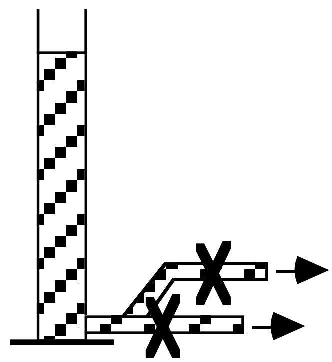
Now apply the continuity equation, where f = v A. What should the velocity look like as a function of radius? Is this qualitatively what you saw? Explain, and discuss with your partners and instructor.

Homework for Monday 9/13

1. A water column is attached at the bottom to a tube with a flow restriction (symbolized by the x) in it. The pressure of the fluid before that flow restriction is 200 Pa. The flow through the restriction is 10^{-6} m^3/sec.



Part a



Part b

a. What is the fluid resistance R of the restriction? You may wish to recall that P = f R.

b. A second tube and restriction identical to the first (and at the same height) is added as shown. (This configuration is called parallel.) Now what is the *total* fluid flow?

c. What is the effective fluid resistance of the two restrictions together in parallel?

2. In the last Guidebook Entry (II.10) you found the radial dependence of fluid velocity where it was free to spread out in two dimensions. Now imagine that we have a source of fluid that then flows out like a fountain, or a mushroom, into three dimensions? How does the flow depend on radial distance? In particular, if the flow is 100 cm^3/sec, and is free to flow out in all upward directions (see figure), what is the fluid velocity at a distance of 20 cm from the source?