Unit 9: Waves in Two and Three Dimensions

In this unit, we will extend the concept of the wave to two and three dimensions. The additional dimensions make for more variety in the modes of propagation, and also allow for a considerable increase in the variety of interference effects that can occur through the superposition of waves.

Session 1: Wave Patterns in Multiple Dimensions

In our previous work with waves in only one dimension, it was always clear that waves originated from a single point along the wave medium, or occasionally from both ends of the wave medium. In multiple dimensions, this is no longer so clear. The source of the wave might be a single point, or multiple points, or a line, or even a full plane.

To help us envision these waves, we will use what is known as a ripple tank; a shallow tank with a glass bottom that allows us to produce waves on the surface of a shallow pool of water. We will follow those by looking at computer simulations of similar waves in multiple dimensions. Unfortunately, we only have seven such tanks, so you will in some cases have to work in larger groups.

Guidebook Entry 9.1: Circular Waves

Fill your ripple tank with enough water to cover the bottom with about a centimeter or so of water. (The tanks are old; watch for leaks!) Let the water settle down, then touch the water surface at the center, and watch the pattern of ripples that develops. Sketch what you see, and describe the time dependence in your own words.

Now run the computer simulation GrayCircleWave, which is in the "2 dim Waves" folder. Does this look qualitatively like what you saw in the water? What differences do you see? Which is more of an idealization?

Since the wave fronts look like circles, such waves in two dimensions are called circular waves. Since we exist in a three dimensional worlds, we more commonly encounter the three dimensional version in which the wave fronts look like spheres. Such waves are not surprisingly called spherical waves, and because they are so common, even circular waves are sometimes erroneously referred to as spherical waves.

Guidebook Entry 9.2: Line and Plane Waves

Again we return to the ripple tank. Take a plastic ruler, and orient the ruler such that its length is parallel to the surface of the water, and the intermediate dimension (i.e. the roughly one inch across the ruler) is oriented vertically. In other words, orient the ruler as if you were measuring a horizontal distance on a wall. Dip the edge of the ruler in and out of the water, so that you make contact along the length of the ruler all at one time. Describe the resulting wave in words, and in a sketch below.

Now run the computer simulation PlaneGrayWave which is in the "2 dim Waves" folder. Does this look qualitatively like what you saw in the water? What differences do you see?

Just as we had with one dimensional waves, we can produce line waves (in two dimensions) or plane waves (in three dimensions) that travel in either direction. Also, by adding together two waves of equal amplitude traveling in opposite directions, we can produce a standing wave. Run the simulation GrayStandPlWave. Describe what you observe. In what sense does this wave move? In what sense does it stand still?

In the next exercise, you will be introduced to the mathematical descriptions of these waves in multiple dimensions.

Guidebook Entry 9.3: Mathematical Representations of Plane and Spherical Waves

The mathematical representation of a circle (or spherical) wave is most easily given in polar (or spherical) coordinates. In either of these representations, the distance away from the center is given by the coordinate r. The oscillating part of the wave is represented using either a sine or cosine function operating on the radial distance r, such as

$$\cos(kr \pm \omega t)$$

where k and ω are defined just as they were for one-dimensional waves.

Describe what the difference is between waves with a + sign after the kr and those with a - sign after the kr.

Which case do you think is more likely to occur naturally?

A plane or line wave is represented much as a one dimensional wave, with the change that it now can have spatial oscillations in x, y, or z. The equation for the wave is written

$$\cos(k_x x + k_y y + k_z z - \omega t)$$

where the three k values give the rate of spatial oscillation in the x, y, and z dimensions. These three k components are often combined into a single vector

$$\vec{k} = (k_x, k_y, k_z)$$

and the plane wave can now be represented more compactly as

$$\cos(\vec{k}\cdot\vec{r} - \omega t)$$

where $\vec{r} = (x, y, z)$.

What is the direction of propagation for a wave represented by

$$\cos(k_y y - \omega t)?$$

What about the plane wave represented by

$$\cos(x + 2y + \omega t)?$$

(Note that it moves in both the x and y directions at the same time, but it is still a plane wave.) For this wave, how far in space must you move in the x direction to go through one cycle of the wave? [Consider a snapshot photograph taken at time t = 0. The crest of one wave is when cos(x + 2y) = 1, or x + 2y = 0. The crest of the next wave is when x + 2y is exactly 2π larger or smaller. So fix y at a constant (zero, say), and see how far you have to move along the x coordinate to get to the crest of the next wave.]

How far must you move in the y direction to go through one cycle of the wave? (Follow the same procedure as above).

Note that the wave crests are a family of lines given by the equation x + 2y = 0, ±2π, ±4π, etc. On an x-y coordinate system, sketch the line x + 2y = 0. Sketch also the line representing the wave crest one cycle later.

From your diagram, find the direction of propagation of the wave (i.e. the angle with respect to the x-axis).

From your diagram, find the wavelength of the wave.

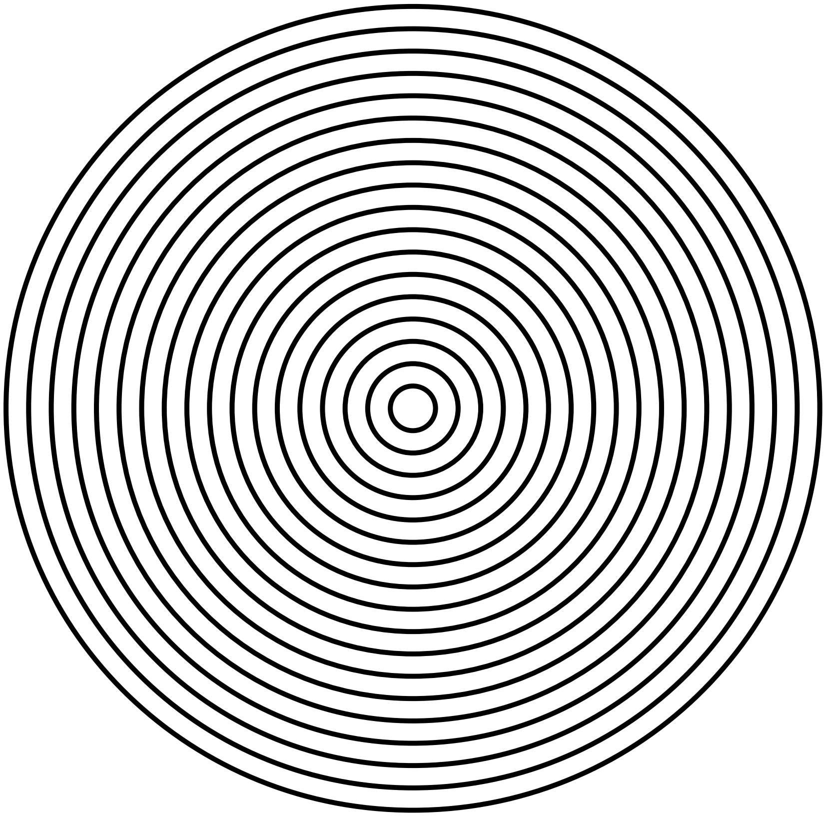
The general form for the wavelength of a plane wave is found from

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

where k = 2π/λ. Does this agree with the value you got for the wavelength in the example above? Discuss any discrepancies with an instructor.

Now imagine that your diagram represents a moving wave that has gone a distance of one wavelength λ in a time equal to the period T. Knowing that T = 2π/ω, and that k = 2π/λ, find the velocity of the wave.

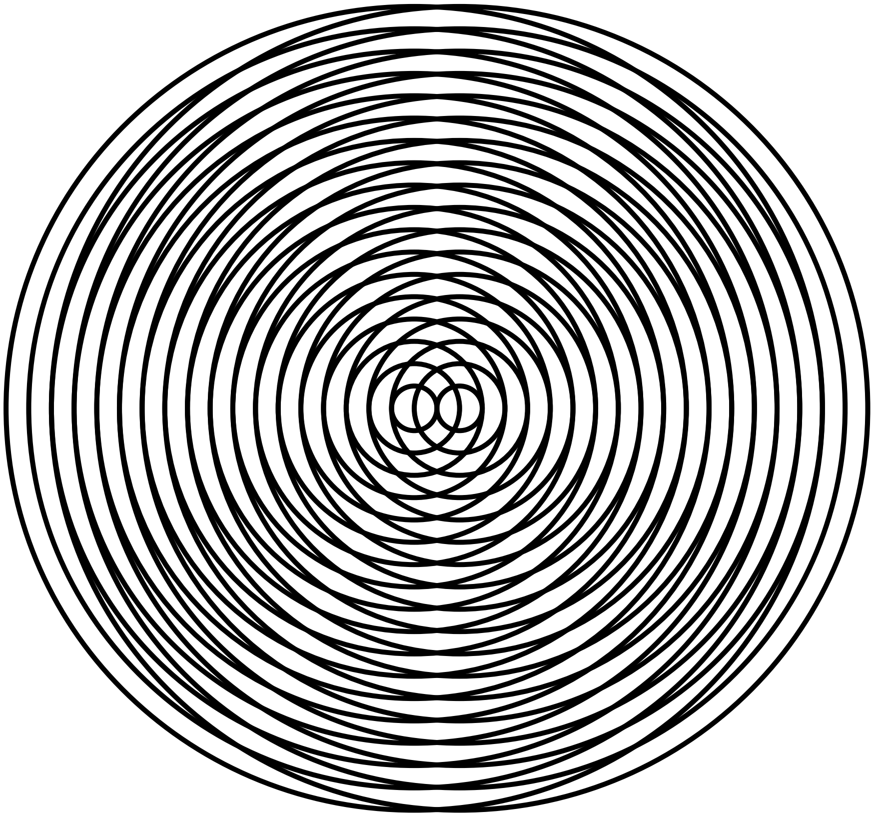
Finally, we want to see what interference effects result when you superpose two circle waves with the same wavelength. In this exercise, we will look at a schematic diagram of a set of circular wavefronts as shown on the following page.



In this instance, you may consider the dark lines to be the wave crests, and the light spaces in between to be the troughs. The waves reinforce one another when crest lines up with crest, and trough with trough. The two waves cancel one another when crest lines up with trough.

Guidebook Entry 9.4: Interference of Two Circle Waves

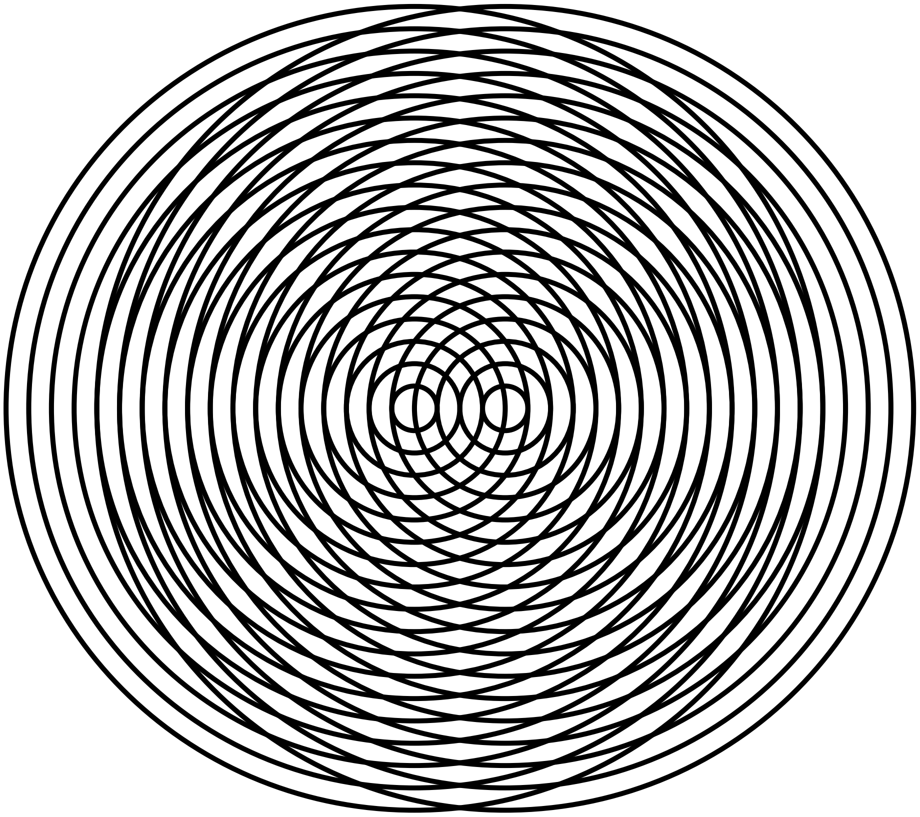
Below you will see two circle waves superposed.



How far apart (in terms of the wavelength) are the two sources (i.e. the centers of the wave patterns?

Mark with lines the regions of constructive interference.

Now consider this set of waves.



How far apart are the centers of these two oscillations? Are they closer or farther apart than the previous example?

Now draw in the lines of constructive interference on this graph.

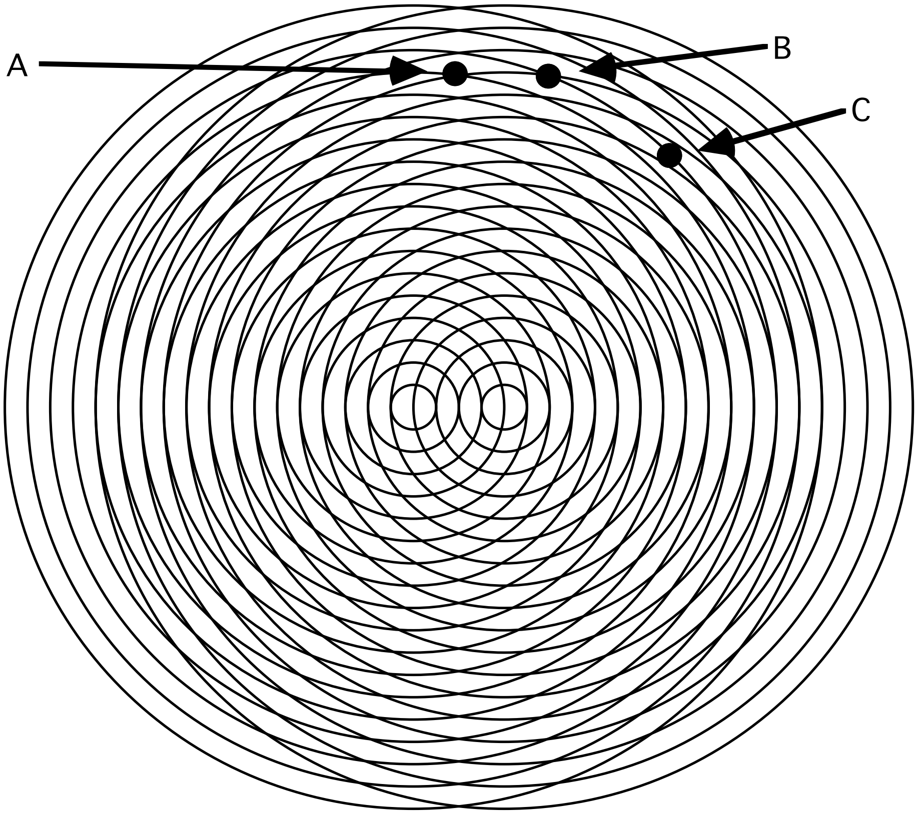
Are the angles between the lines of constructive interference closer or more spread out than in the other example? Can you state a general rule for this?

Session 2: Interference and Diffraction

In the last session, we introduced the example of interference between two waves from two different point sources. In this session, we will investigate this in greater detail, and observe the interference that results when light passes through closely spaced slits. This allows us to see that light does indeed have wave properties, and gives a particular clean example of waves for analyzing interference phenomena.

Before we look at experiments, lets take the time to do some mathematical analysis of the interference patterns you looked at last time.

> *Guidebook Entry 9.5: Mathematical Analysis of Interference with Two Point Sources.*
>
> Below is a duplication of a figure from the previous session. The lines have been lightened so that you can more easily draw on the figure.



> Consider point A. Is this a point of constructive interference, or destructive interference? Explain.

In terms of the wavelength, what is the distance from the left wave source to point A?

In terms of the wavelength, what is the distance from the right wave source to point A?

Now consider point B. Is this a point of constructive or destructive interference?

In terms of the wavelength, what is the distance from the left wave source to point B?

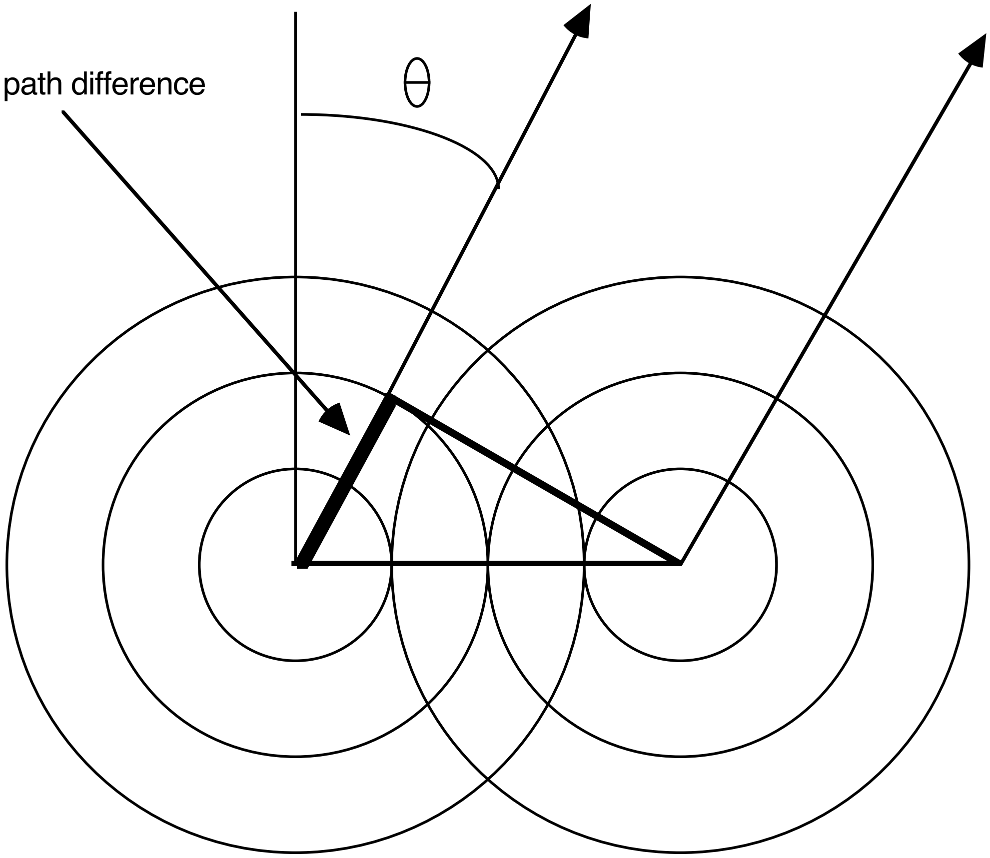
In terms of the wavelength, what is the distance from the right wave source to point B?

Now, consider the point C. Is this a point of constructive or destructive interference?

In terms of the wavelength, what is the distance from the left wave source to point C?

In terms of the wavelength, what is the distance from the right wave source to point C?

The figure below shows a magnified view of the region near the source of the waves.



The boldest line marks the difference in distance from the two sources to a very distant point (in the general direction of points B and C of the figure on page 9). If this is actually the path difference, do you expect the distant point to be a point of constructive or destructive interference? Why?

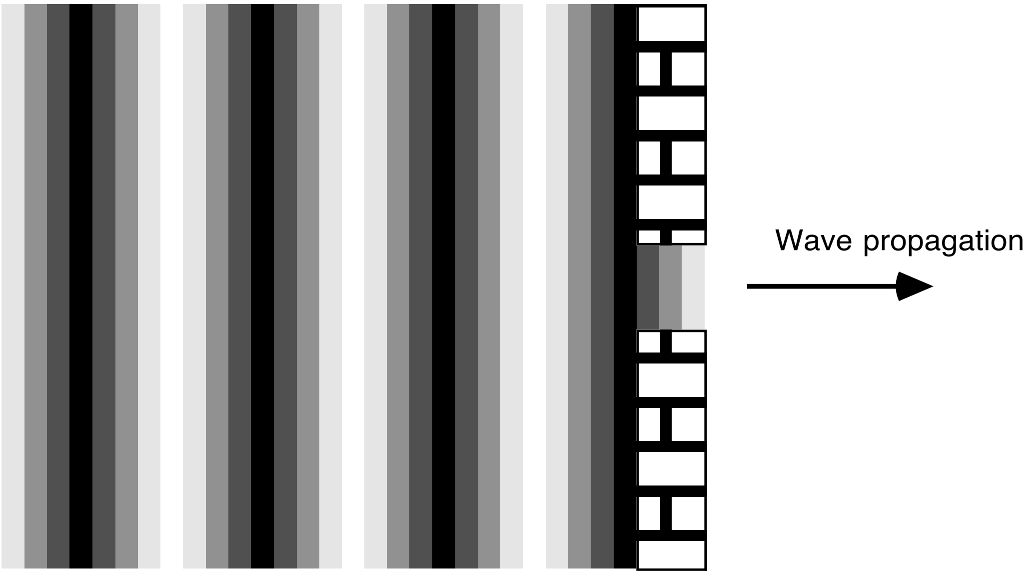
Can you give a general requirement for the path difference in order for constructive interference to occur?

Notice that the triangle marked in bold lines is nearly a right triangle, and that the one leg always gives the path difference. What requirement can you place on the angle to give constructive interference? Give your result as an equation. Use the variable "d" to represent the distance between the two wave sources. Check with an instructor after you have a result.

The most common way to achieve this type of an interference effect is not in fact by looking at two point sources, but rather by letting a plane wave fall on a barrier that has two gaps in it. First, however, it is important to see how plane waves behave when they encounter a barrier with a narrow gap in it.

Guidebook Entry 9.6: Plane Wave Incident on a Barrier with a Gap

The figure below is meant to indicate a plane wave (indicated by the varying shades of gray) incident on a barrier (the brick wall) with a gap in it. Notice that the size of the gap is comparable to the wavelength of the wave.



What do you expect will happen to the wave once it passes through the gap?

Do this experiment with one of the ripple tanks. Make sure that the barrier has a gap that is comparable to the wavelength of plane waves that you can produce with a ruler. Sketch the resulting waves on the far side of the barrier.

Since plane waves incident on a barrier with a gap in it look just like half spherical (or circle in 2 dimensions) waves, this provides an easy way of generating two or more point-like sources of waves. We simply allow plane waves to hit a pair of slits in a barrier. Because the waves come from the same source, you can be assured that they are of the same frequency, wavelength, and maintain the same phase relationship between one other. The interference pattern that results is called a double slit diffraction pattern. It has maxima at angles determined by the relationship you discovered earlier,

$$d\sin\theta = n\lambda$$

where d is the spacing between the slits, n is an integer describing how far away the maximum is from the central maximum, and λ is the wavelength of the wave.

In the next activity, you will observe diffraction patterns from light, which both indicate the wave nature of light, and allow us to see clearly the pattern that results from double slit diffraction.

Guidebook Entry 9.7: Double Slit Diffraction of Laser Light

Laser light is particularly useful for diffraction experiments. It has a constant, well-defined wavelength, and is all traveling in the same direction, and therefore is well approximated by a plane wave.

Position the laser beam on one of the double slits on the glass plate. Describe the pattern that the laser beam makes on the wall after passing through the slits.

Use one of the key sheets to find out how far apart the slits are. Then deduce the angle between the central maximum and the next maximum, and from that calculate the wavelength of the laser beam light. (This is a multi-step process, so check your result with an instructor.)

Now choose one of the other pairs of slits to pass the laser through. From this diffraction pattern, see if you get a wavelength that agrees with your previous value.

In the final activity, you will see what happens when we increase the number of slits.

Guidebook Entry 9.8: Diffraction from Multiple Slits

Now shine the laser beam on the series of slits that maintains a constant spacing between the slits, but has more and more slits.

What happens to the relative separation of the two adjacent maxima as the number of slits increases?

What happens to the character of the maxima as the number of slits increases.

Place the cardboard slide in front of the laser beam. This is a diffraction grating, with thousands of tiny slits adjacent to one another. Since the slits are so close together, the first maximum is at a considerable angle off center. Locate this maximum, and together with the laser wavelength (which is very nearly 633 nm, or $6.33\text{x}10^{-7}$ m) calculate the spacing between the slits. How does this compare with the value printed on the slide mount?

Since the diffraction process is wavelength dependent, what do you expect will happen if you shine white light (which is a mixture of many wavelengths) on the grating, or equivalently, look through the grating at a white light?

Try looking through the grating at lights. Describe what you see.

In a later unit, we will use this property of the diffraction grating to separate light into its constituent colors to examine the spectrum of colors present in light.