Unit 8: Waves

In this unit, we finally give some recognition to the fact that the motion we have been describing does not happen in isolation, but in fact often has significant effects on distant objects. In particular, we extend the concepts associated with oscillations to the transmission of these oscillations to distant objects through a type of motion that is generally referred to as wave motion. We apply this to the motion that we conventionally think of as wave motion (e.g. water waves) as well as many phenomena that we can't directly perceive as wave motion (e.g. sound and light). Equally important is the startling result from quantum mechanics that even material objects, when viewed on an atomic scale, exhibit wave behavior. The study of classical waves is then essential if we wish to be able to develop an intuition for quantum mechanics.

Session 1: Introduction to Wave Motion

In this session, we will simply develop some familiarity with the basic features of wave motion, including some terminology and some of the mathematical description of the waves.

In this first activity, you will observe some wave motion first hand, and describe the qualitative features in your own words.

Guidebook Entry 8.1: Types of Wave Motion

To help you visualize wave behavior, we have some extra-long "slinkies" which have nice, slow wave propagation that is easy to watch.

With a group member at each end, stretch a slinky out along the floor to a length of two to three meters. With your partner holding her end still, push your end quickly toward your partner then back again to its original position. Describe what you see:

Now take the end of the slinky, and slide it quickly to the side (perpendicular to its length) and then back again to its original position. Describe what you see:

These waves are usually referred to as "pulses." They are localized along the spring at any one time. If the pulse is a perpendicular motion of the spring, it is referred to as transverse pulse. If the pulse arises from a compression of the spring, it is referred to as a longitudinal pulse.

Now take the slinky, and rather than a transverse pulse, form a transverse wave by moving the end back and forth in sinusoidal motion (as for harmonic motion of an object subject to a spring force). Describe what you see:

Now take one of the ropes, and hold it in the air between two partners. Let one partner generate a transverse wave along the rope by moving the end from side to side. Then generate a transverse wave by moving the end up and down. How are the resulting waves the same? Different?

The two types of transverse waves you observed above are said to have different polarizations; one has a horizontal polarization, and the other a vertical polarization. One can also generate different kinds of polarizations by moving the end of the rope at a 45˚ angle, or by moving the end of the rope in a circle. Try them!

In the next exercise, you will look at the slinky again, only this time you will use a digitized movie of the motion to get a semi-quantitative view of the motion of waves and pulses.

Guidebook Entry 8.2: Video Analysis of Wave Motion

Open the digitized movie called "Pulse" that is on your computer. Move ahead to a convenient frame where you can see the pulse clearly. Sketch what the pulse looks like at this snapshot in time:

Now choose some point on the spring, that is, a fixed left/right position on the computer screen. Step the movie along frame by frame, and record the displacement of the spring (i.e. how far it is off of the black line) as a function of time. (Each frame takes 1/30 sec.) Graph these data, and either sketch or attach your graph.

Frame Number	Displacement

Compare the pulse as a function of position (your snapshot) and the graph of the displacement as a function of time at a fixed point along the spring. They should look very similar. Do they?

This is a characteristic feature of wave motion: the variation in space looks just like the variation in time. Let's see if this occurs in practice with wave motion as well as the pulse. Open the movie called "Wave." Sketch a snapshot of the wave below:

Now repeat your measurement at a point along the spring as a function of time for this wave. Graph and attach or sketch your result.

Frame Number	Displacement

Does the wave in time look just like the wave in space? How are they the same and how are they different?

Just as the size of an oscillation is called the amplitude, the size of a wave is also called the amplitude. Why does the wave amplitude get smaller as it travels along? How do you think you could fix this, or what other wave system do you think might not exhibit this trait?

The wave shape appears to travel along the spring. What is the velocity of the wave? Describe how you measured this.

Now go back to the pulse movie. What is the velocity of the pulse? Is it about the same as that of the wave?

The feature of similar variations in space and time is easily incorporated into a mathematical description. If the "snapshot" function of the wave is given as

$$f(x)$$

then the wave can be described in both space and time as

$$f(x-ct),$$

where c is a constant. In the next exercise, you will examine this in more detail for some specific functions. In general, we write wave functions in a more general way, as

$$f(kx-\omega t),$$

which allows us to use sinusoidal functions more generally. The next activity encourages you to look at both ways of denoting wave motion.

Guidebook Entry 8.3: Mathematical Analysis of Wave Motion

Use Excel to graph the function

$$y = \frac{1}{\left(x^2 + 1\right)}$$

from x= -10 to 10, and show it is a reasonable function for a pulse. Sketch or attach your graph.

Now create a mathematical expression for a moving pulse by replacing x with x-ct. Graph this with Excel for t = 0 and t = 1 with c = 4. Sketch or attach your graphs.

What is the wave velocity of this pulse, that is, how far the peak moves per unit time? How does this relate to the constant c?

We found that a general expression for a sinusoidal oscillation was expressed as f(t) = sin(ωt). We can generalize this to a form that includes both the space and time variations as f(x,t) = sin(kx-ωt).

Show that this f(x,t) moves by making a graph of f(x,t) versus x at two different times. Choose at least initially k=2 and ω=5. Describe what you observed.

What is the velocity of the wave for your particular case?

Can you find a quantitative relationship between your observed velocity and k=2 and □=5? Does this same formula work for the x-4t substitution you made on the previous page (first, you need to answer what are the value of k and □ in this case.)?

Write the general relationship between wave velocity, of k and □. Check your result with an instructor.

The wavelength, usually symbolized by □, is the distance from one crest of a sinusoidal wave to the next (or any two identical features). Can you find a mathematical relationship between the constant k and □?

Session 2: Reflections, Superposition, and Standing Waves

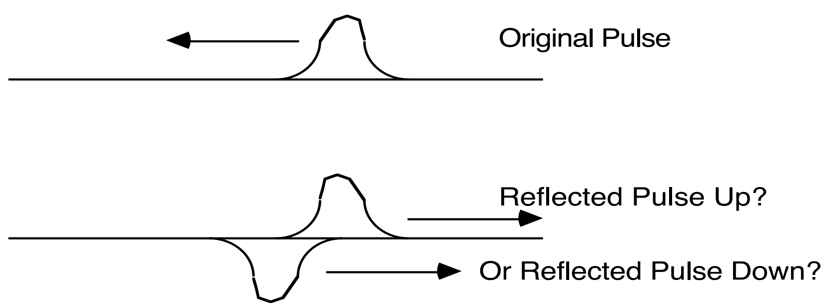
In this session, we will deal with what happens when two waves overlap with one another. In contrast to solid objects, which bounce off each other, two waves obey a principle known as superposition, in which the two waves add to one another.

In this first exercise, you will observe what happens to a wave pulse when it reaches the end of the wave medium.

Guidebook Entry 8.4: Observation of Reflected Pulses

Take one of the long slinkies, and stretch about half of it along the floor (keep the remainder coiled together) with a partner at either end. One partner sends a transverse pulse down the slinky while the other partner holds the other end still. Describe what happens after the pulse hits the far end. (You may need to practice a while to get a good pulse that travels with considerable amplitude all the way to the end. Don't hesitate to ask for help.)

Does the reflected pulse seem to go in the same direction as the original (say an upward bump), or in the opposite (as a downward bump)?



In general, when a wave or a pulse reaches the end of the wave medium (or even a sharp change in the character of the wave medium, such as where a heavy rope is connected to a light rope) a reflected pulse is generated. In most (but not all) cases, that reflected pulse is inverted, as you should have observed with the slinky. We will return to this phenomenon shortly, but first we must understand a basic principle of the interplay of two waves, known as superposition.

Guidebook Entry 8.5: Superposition

Imagine that we have two wave pulses traveling in opposite directions along the slinky, started from either end. What do you predict will happen when they "collide" with one another?

Stretch a long slinky out on the floor, and perform the experiment. Take care to generate pulses that are both "upward." What do you observe?

Do you think the pulses bounced off of one another, or did they seem to pass through one another?

You should be able to give a more definitive answer to the last question by sending an upward pulse from one end and a downward pulse from the other end. Do this. Do the pulses bounce off, or pass through? If you find it hard to tell, ask an instructor for help.

Finally, send two upward pulses toward one another, and pay special attention to the pulses as they pass through one another. Is the combined pulse larger or smaller than the individual pulses at that point? (It may help to do this close to a wall so it is easier to see the amplitude of the combined pulses.)

The behavior of waves in the presence of other waves is very simple: each wave or pulse continues as if the other waves were not there at all. The total wave is simply the algebraic (i.e. including + or - signs) sum of the original waves. That is why the combined pulse was twice as big as the individual pulses as they passed through one another. We can turn this property around, and use it to describe more complicated situations by breaking a complicated wave pattern up into simpler pulses and waves. In the next exercise, we will use this technique to explain why we found the reflected slinky pulses came back inverted.

Guidebook Entry 8.6: Excel Model of Pulse Inversion

I have created an Excel spreadsheet called "Pulse Reflection" which you will find in the "1 dim Waves" folder on your computer desktop. In this spreadsheet, I have used a handy trick. I have designated a single cell as the time. All of the cells that need to know the time are referenced to this cell. To make sure that this reference did not change as I "filled down," I made the reference to this cell "absolute" by putting a $ in front of the number part of the cell reference. You should look at one of the cell entries, say the one in B10, to check on this. I also used the same pulse function

$$y = \frac{1}{\left(x^2 + 1\right)}$$

that you used earlier in this unit.

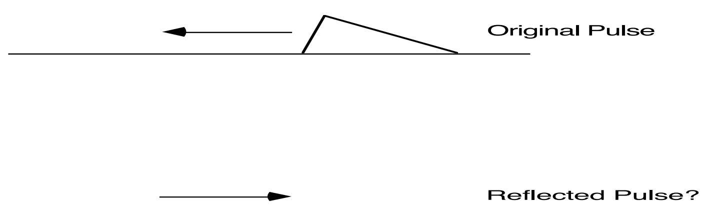
We have required the end of the spring to remain motionless. We do this by applying forces that end up generating the reflected pulse. The correct solution for the reflected pulse will be the one that keeps that end motionless. In our Excel model, we do this by imagining the incoming pulse as traveling off the end of the slinky into an imaginary region where the slinky isn't, and the reflected pulse coming in from that imaginary region. The total solution is simply the superposition, the sum, of these two pulses. This sum is only given on the right half, where the slinky actually exists, and is shown in blue markers. Start with the time value at -7. Increase the time by typing greater values (increase by one at a time) followed by the enter key.
What color curve represents the incoming pulse?

What color curve represents the reflected pulse?

Increase the time enough so that the two pulses overlap significantly. What is the value of the sum wave at the end of the slinky, that is, x = 0?

Increase the time by .5 at a time. Does the value of the sum wave at x = 0 ever vary? Explain.

Imagine that the incident pulse had been asymmetric, like the figure shown. What would the reflected pulse look like? Explain your reasoning, and check with an instructor.



If pulses are reflected from the end of a wave medium, then one would certainly expect a sinusoidal wave to be reflected as well. So, to understand what wave motion results when sinusoidal waves hit an end, we must investigate what the superposition of a left-going wave and a right-going wave looks like. The superposition of these two waves gives rise to a phenomenon known as standing waves.

Guidebook Entry 8.7: Excel Model of Superposition of Traveling Waves

Open up the Excel file "Standing Wave" from the "1 dim Waves" folder. This file is very similar to the pulse spreadsheet file you just worked with. Examine some of the cell entries to make sure you understand how the spreadsheet works; ask an instructor if you have any questions.

Set the time to 1, and then increase the time value by 1 at a time. Do the component waves move as they are labeled?

Now look at the sum wave. Is it moving to the left, or to the right? Discuss your answer with an instructor.

Describe in your own words what a standing wave oscillation looks like:

What is the value of the wave at x = 0 as a function of time? Points like this are called nodes.

Explain why a standing wave pattern on a rope must have a node at the end that is held motionless.

If x = 0 is the fixed end of a rope, and hence the origin of the reflected wave, is the wave reflected from this point inverted relative to the original incoming wave? Explain.

We can also investigate standing waves analytically. We are adding together two waves, one expressed by $\sin(kx - \omega t)$ and the other by $\sin(kx + \omega t)$. Which one of these goes to the left, and which to the right? Check your answer by peeking at the formulas in the Excel spreadsheet, or checking with an instructor. Can you explain in words why each moves as it does?

To add these two sign functions and produce a mathematical form that more clearly shows the standing wave phenomenon, we need the trigonometric identity

$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right).$$

Use this formula now with $\sin(kx - \omega t) + \sin(kx + \omega t)$. You should, with not too much algebra, get an expression for this wave that one can describe as a sine wave in space (i.e. just a function of x) multiplied by an amplitude that oscillates in time. Find this, explain it, and check it with an instructor!

Guidebook Entry 8.8: Observation of Standing Waves

Now that you have seen a theoretical description of a standing wave, try observing one experimentally. Since we want to see a good reflected wave, you must use a rope that does not suffer the damping of the wave that the slinky on the floor does. One partner holds one end of the rope motionless, while the other partner moves the other end back and forth long enough for the reflections to build up along the whole rope. Can you get a standing wave pattern?

Do you see nodes? How many?

Try shaking the end of the rope more rapidly (i.e. at a higher frequency). Can you get the number of nodes to change?

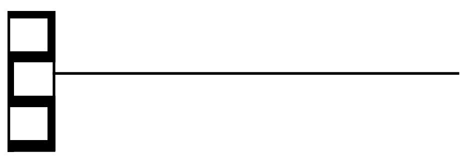
Session 3: Sound and Resonance

In this session, we will use the concept of standing waves to discover the phenomenon of wave resonance. In the process, we will also learn a little bit about sound, and see some computer displays of waves that will allow us, in the following units, to visualize waves in two and three dimensions.

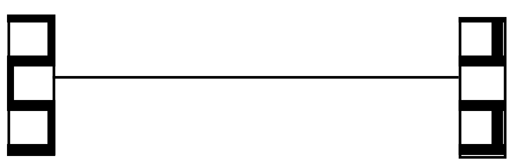
In the last session, we observed standing waves by taking a rope and wiggling one end and holding the other end fixed. This gave us a requirement for the end of the rope (what mathematicians and physicists call a boundary condition) that the value of the wave there must be equal to zero. In the next exercises, we will consider what happens when we demand that *each* end of the rope must be held motionless.

Guidebook Entry 8.9: A Rope with Two Fixed Ends

We saw that with one end of the rope fixed (i.e. held motionless), we found that whatever wave was sent in to that end was reflected in an inverted fashion. When the incoming wave was sinusoidal, this resulted in a node (a point in the rope that always stays at zero displacement) in the waveform at the end. Sketch what this typically looked like below:



Now imagine that we have attached the rope firmly to walls at either end. This means that the rope must not move at *either* end. Sketch what a possible standing wave would look like.



Give a general statement about where nodes must be for a vibrating rope or string with both ends held still (fixed).

In this last exercise, you made a guess about where the nodes were for a string with both ends fixed. You should have at least stated that there must be a node at each end, and may have made some speculation about where other nodes might lie. To give us some theoretical guidance about the placement of these other nodes, we must do a little more detailed investigation of standing waves.

Guidebook Entry 8.10: Mathematical Digression on Standing Waves

We found that a standing wave resulted when we superposed two traveling waves in either direction that had equal wavelength and amplitude. What was the spacing between nodes that you saw in terms of the wavelength of the original wave? You should refer to your Excel results and the mathematical form from the last session (G.E. 8.7)

Assuming that the velocity of propagation ω/k for the wave does not vary, how does the spacing between the nodes change if you double the frequency of the wave?

Most typically, we constrain the length (by attaching the rope or string to walls or some other mount) and the velocity (through tension and rope weight) of a vibrating rope or string. Find the mathematical expression for the resulting allowed frequencies in terms of the length l and wave velocity v. Check your result with an instructor.

In the next activity, you will use the fact that sound is a longitudinal wave (much like the longitudinal pulse you sent along the slinky) that results when the air is vibrated by some object, such as a tuning fork, a speaker, or your vocal cords. We perceive the frequency of the sound wave as the pitch of the sound. In this activity, you will use a taut string as the vibrating element, and use your sense of pitch to qualitatively detect changes in frequency.

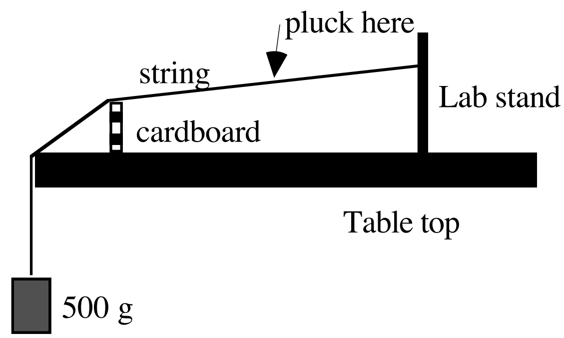
Guidebook Entry 8.11: Listening to Standing Waves

A vibrating string, such as a piano or guitar string, vibrates primarily in what is known as its fundamental mode. This means that the only nodes of the wave pattern are at either end. If this is true, and the wave velocity in the string (which depends on the tension of the string and the mass of the string) is 500 m/sec, what is the frequency of vibration of a string of length 60 cm?

How will the frequency change if you cut the length to 30 cm?

What do you expect to be the same for the resulting sound wave: will it have the same wavelength or the same frequency or both as the wave on the string? Why? Discuss this with an instructor.

You should have predicted that the frequency (of both the sound and the string) goes up. This is a general feature of vibrating strings; the pitch increases as the string is made shorter. This is the primary method of changing the pitch on stringed instruments such as violins and guitars. The fingers are used to adjust the length of the vibrating string. You can build a primitive version of a stringed instrument using a string, a 500 g weight, and a piece of cardboard. Set up your instrument as shown in the following figure.



Listen closely to the string as you pluck it somewhere near the middle. Listen to the pitch as you move the cardboard around so as to change the length of the string. How does the pitch seem to depend on the length of the string?

Guidebook Entry 8.12: Listening to Overtones

We can force a string to vibrate in a way that places one, two, or more nodes in between the ends. If the frequency of the mode with no nodes is 200 cycles/sec, what is the frequency of the mode with one node between the ends?

Two nodes between the ends?

Musicians as well as physicists refer to these higher modes of vibration as harmonics. We have brought in a guitar since that provides an easy source of the harmonics. We will look at the sound wave from a microphone (which converts the sound wave into an electrical signal) on an oscilloscope, which is like a very fast version of Mac Motion for looking at electrical signals. If one of you is a musician, you can find the harmonics yourself, or ask an instructor if you are not sure what to do. Measure the period and calculate the frequencies of the sound waves of the:

fundamental:

first harmonic:

second harmonic:

Are the ratios of the frequencies the same as what you predicted just before? Discuss any discrepancies with an instructor.

Guidebook Entry 8.13: Computer Models and Waves in a Circle

We will soon be dealing with waves that we cannot easily see or hear. To help us visualize them, we will use computer models which you will find in the "1 dim Waves" folder which you used last session.

First, run the sine/graywave program. This shows a traditional sine curve, and a grayscale version beneath it. The maxima are represented by bright spots, the minima by dark spots. Click the mouse button to stop the program when you are through (it will mess up the color of some of your windows; close them and reopen them to straighten them out).

This could be used to represent a traveling wave. Start the program "1dGrayWave" to see what this would look like.

Now, if you had a standing wave made from this model, what shade would the nodes be?

Start the program "1dGrayStandWave." Identify the nodes, that is the points that don't change their shade of gray. Were they what you expected?

We found that we could have a stable oscillation on a fixed length of string only if we put nodes at either end (that is, had an integral number of half wavelengths along the string). Now imagine that we had waves constrained to move along a circle. To have a well defined wave, the wave would have to match up again after it moved all the way around the circle. In other words, it would have an integral number of *full* wavelengths around the circle.

If you had a circle of radius 5 cm, what would the three longest allowed wavelengths be for waves traveling around that circle?

Open the program GrayHoopWave. How many wavelengths are there around the circle in this simulation? Sketch what the figure looks like below.