Unit 2: Freefall, Forces and Acceleration

In this unit, we will attempt to rediscover the special role that acceleration plays in motion by looking at freely falling objects. We will use both the motion detectors and video analysis to observe these phenomena. First, we will look at a very simple and common motion: that of freefall. After this, we will consider our conception of force, and see how that relates to motion. Finally, we will formalize this in Newton's second law, which is the cornerstone of the study of motion, the subdiscipline physicists call mechanics.

Session 1: Video Analysis of Freefall

Why do things fall down? Naturally, we all know things fall down because gravity is pulling on them. This pull is remarkably uniform and unchanging. We understand this pull as our weight, which seems not to depend on whether we are indoors or outdoors, upstairs or downstairs. If this gravitational pull is simple and constant, we would hope that there is something correspondingly simple and constant about the motion that results from gravity. The next activity will help you discover some of those simple features of freefall.

Guidebook Entry II.1: Guessing the Distance Graph for Freefall.

Let's think about what it is like to fall. Imagine that you are jumping off a tree limb that is two meters above the ground. Sketch below your qualitative guess of what your distance-above-the-ground-versus-time graph would look like.

Now perform the following experiment. Have one of you in your group close your eyes. Take a ball, or a bean bag, and drop it (from rest--just release it) from a height of about 10 cm above the floor. Now drop it from a height of 2 meters above the floor. Can the person with her/his eyes closed tell which drop is which? (You may want to randomly switch the order to make a fairer test.)

What causes the different sound when the object hits? In which case would you guess the object is moving faster when it hits the floor--the high drop, the low drop, or are they the same? To help you think about this, could you fool the listener by *throwing* the object from the short distance?

If we can interpret this as saying that the speed of an object increases as it drops, how should the *slope* of the distance versus time graph behave as time goes on? In other words, should the [absolute value of the] slope of the graph be greater at the left (earlier times) or at the right (later times)? Explain.

Given your answer to the last question, look back at your graph of distance versus time for a falling object. Do you like your graph, or do you need to adjust it? Sketch an improved graph below if you are not fully satisfied with your previous one.

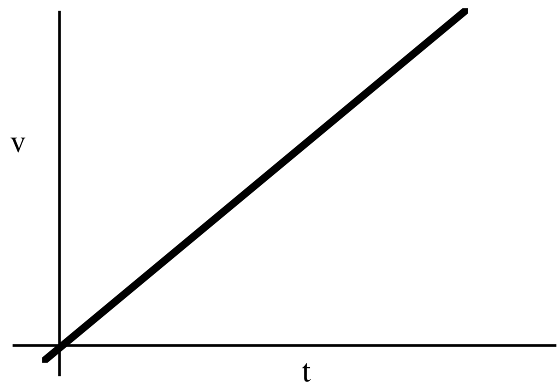
Recall that we are looking for some quantity that describes the motion that is constant, to go along with our constant gravitational pull. Could distance be this constant? How about velocity? Why or why not?

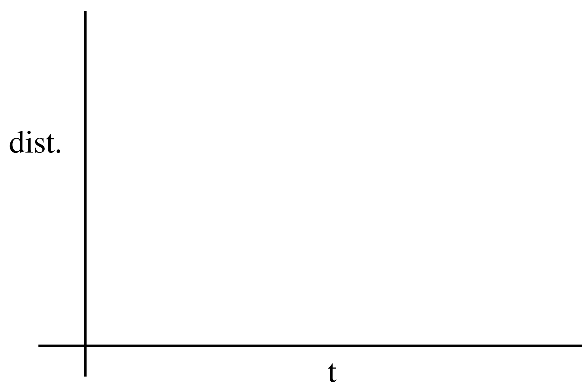
Given the answer to the last question, what is the *simplest* possible graph that one can have of velocity versus time that is consistent with the qualitative behavior of velocity as a function of time? Discuss this with your partners, and with an instructor.

Hopefully in the last section, you decided that a linear function of velocity versus time was the simplest possibility. In this next activity, we will examine what significance this has for the position-versus-time function.

Guidebook Entry II.2: Applying Some Calculus to Freefall

Let's imagine that we have the following function of velocity versus time. Make a qualitative sketch of what position versus time might look like consistent with the velocity function. Explain your answer.





Can you tell what the value for distance at t=0 is? Why or why not? Discuss this with your group, and check with an instructor.

The [instantaneous] velocity versus time function is the time derivative of the distance (or position) function. In an equation, this would be expressed as

$$v = \frac{dx}{dt}.$$

Consider the velocity versus time graph again. Can you write a mathematical expression for velocity versus time that is consistent with this graph?

$$v =$$

What new function would have a function like your velocity function as its time derivative? If this is confusing, recall what we did in the last class with Excel finding velocity functions from position functions. Did you deal with a function whose derivative was a straight line?

If the velocity is not itself constant, what feature of the velocity graph *is* constant?

You should now be hot on the track of what Galileo first discovered: the special, constant thing about freefall motion is *not* the first derivative of the motion (velocity), but rather the derivative of the *velocity* curve. Mathematicians would call this the second time derivative of the position function, and physicists give this the name acceleration:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}.$$

If we release an object from rest, it starts with zero velocity, and the velocity increases steadily in a linear fashion. This then implies that the position function looks like

$$x(t) = \frac{1}{2}at^2 + \text{constant} .$$

[Why did I need to add the constant term? And is the acceleration "a" positive or negative for freefall?]

In the following exercise, you will take some data by hand to try to get a feel for how plausible our constant acceleration model is for freefall, and to get an idea of how big this constant acceleration is.

Guidebook Entry II.3: Drop Your Socks!

We have a great number of bean bags carefully made from baby socks and dried beans. Grab a couple of these and a stop watch, and measure how long it takes a bean bag to fall various (at least four) distances. Measure each height a couple times by a couple different timekeepers. It also helps to have the same person drop the bag and keep the time, so that the initial release is most accurately timed. If you want some greater distances, the west stairwell (by the exit nearest ARH) has some good height. It measures 4.39 meters from the basement floor to the top of the open metal railing on the first floor, and 8.28 meters from the basement floor to the top of the railing on the second floor.

height	time1	time2	time3

Graph your data as distance versus time using Excel, and attach your graph. Make sure this is an x-y graph! Is the t^2 dependence a plausible one? Explain.

You probably didn't get the same time for each one of your measurements at any one height. Why might this be? Is gravity changing? How accurate are your measurements?

From your data, can you give a guess of how big the acceleration of freefall is, assuming it is a constant? How well do you believe your number? What is the sign of this acceleration? (You might want to check with an instructor on these points, since they can be rather subtle.)

Now we want to take somewhat more accurate measurements of distance versus time in freefall. We will use video technology to allow us to do this. Your computer has a digital video of a falling beanbag—called freefall.mov. The video frames are taken at 1/30 sec time intervals. We include a meterstick in our videos to get the scale right, which must be near the dropping object. You need to analyze this movie frame by frame. To do this, you simply step through the video frame by frame using the left/right arrow keys on your keyboard, and mark the position of the dropping ball or bag on your computer screen with a washable marker, and then measure with a ruler to find position versus time. There are two copies of the video; the second is horizontal which looks strange, but gives more accurate results since it uses more of the frame. Use whichever you choose, but do enlarge the viewing window to fill the monitor.

Guidebook Entry II.4: Analyzing a Video!

The rough technique for this is described above. Our video has a 2 meter scale shown in the movie. Use this to calculate a conversion factor that converts distance on the computer screen (a few cm) into real distance (up to 2 m). Make your measurements of distance from the release point, and then convert them to actual height (in meters above the floor) using your scale factor.

time	scale height	real height

Now use Excel to graph these data. Attach the graph. Does the t^2 dependence look plausible?

Now take your data and calculate the average velocity over each 1/30 sec time interval. You can do this by remeasuring the distance between adjacent points, or by taking differences between points using Excel and your table of position data. Either way, graph your results. Does the velocity seem to have a linear dependence on time?

How does the acceleration relate to the velocity graph? How nearly constant is the acceleration?

What value do you get for the acceleration from this velocity graph? How does that compare with your earlier rough measurement from dropping socks?

You might also try fitting your height versus time data with a polynomial fit in Excel. Click on your data points in the graph until they are highlighted. Then choose the trendline item from the insert menu. Make sure to choose the "show equation on graph" item from the options list. Ask for help if you need it! Compare your acceleration value to previous ones.

Session 2: Measuring Forces

In this session, we will develop some of the concepts of force and mass. Fortunately, the conventional meaning of both of these words is closely related to the technical physics meaning of them, so there are not a *lot* of bad misconceptions. Nevertheless, it is important that we know what we are talking about!

A force is simply a push or a pull. Conceptually this is not difficult to understand, although it is not obvious how to measure this quantitatively. It is a remarkable accident (or perhaps revealing coincidence) that the several most obvious ways to measure force are equivalent. We will rely initially on some force scales that are calibrated springs.

Mass is a bit more subtle. It is a way of measuring the amount of material that we have. The easiest way to measure this is by the weight (force of gravity) of the object of interest, dumping the definitional burden back on forces. Unfortunately, this *measurement* method is not the same as a definition of mass, and leads to considerable confusion between mass (amount of stuff) and weight (force of gravity). For in deep space, the weight goes away--there is no gravity--but the stuff clearly does not, so presumably neither does the mass. In any event, we definitely want the quantity to be what is called extensive; in other words, the total mass of two objects is simply the sum of the masses of each one.

Guidebook Entry II.5: Spring Scales and Weight

The mass, or amount, of material can be gauged by the weight. The weight is the force that gravity exerts on the mass. We can feel this weight by holding the mass up. As a simple exercise, take two masses, one larger than the other. Close your eyes, and have one of your partners place the masses sequentially in your hand. Can you tell which one is which?

Now take one of the spring force scales. The units of force in the MKS system are Newtons, abbreviated N. Each group has a brown force scale to allow you to measure force. Pull on this gently to see what 1 N, 5 N, and 10 N of force feels like. Then try without looking to pull with a force of about 5 N, and have a partner check the scale. This is nearly one pound (the weight of half a kilogram). Have each of your partners try. What were your results? Who came closest?

Now use the force scale to measure the amount of force necessary to hold up several weights; at least 200 g, 500 g, and 1 kg. Make a table of force versus mass, and then graph that, and attach the graph.

How does force of gravity depend on the amount of mass? Can you express this relationship as an equation?

As you might have guessed, we also have a computerized method of measuring forces, which the developer of this device calls a force probe.

Guidebook Entry II.6: Using the Force Probe

To use the force probe, start up the LoggerPro program. This should open three graphs, Force, Position, and Velocity. The force probe looks like a black box with a hook on the end of it. Before you can use it effectively, you need to calibrate it. To do this, choose the Calibrate item in the Experiment menu, choosing the Dual Range Force Probe. This brings up a calibration window. With no force on the hook of the probe, enter a zero as the force in the first force box, and click the Keep button. Now pull down on the hook with say 5 Newtons using the brown spring force scale. Enter 5 in the second force box, and click Keep, and then close the box by clicking the Done button. Then start the data acquisition, and wiggle gently (a few Newtons worth of force) on the force probe. Does the force graph do what you expect? Which direction is positive force?

Now use the force probe to measure the weight of the same masses you used before. How do your values compare to your previous results?

We also know that forces can make things go. In the next exercise, you will get a first, qualitative exposure to the relationship between force and motion. In the next session we will do this quantitatively with the computer, but it is important that you develop a good intution about the relationship among force, mass, and motion.

Guidebook Entry II.7: Making Things Go!

For this exercise, you will apply a nearly constant force by pulling on carts with a spring force gauge applying one Newton. Try this for a cart with a one kilogram weight on it. Ask for help if you can't keep the force very constant.

We have a number of 1 kg weights to allow you to change the mass of the object you are applying the force to. Start the cart at rest with 2 kg on it. Apply your 1 N force to the cart on the floor or your table (initially not moving) for 1 second. What happens?

What happens if you apply the force for 2 seconds? How does the motion compare to what happened when you applied the force for only 1 second?

What difference do you expect in the resulting motions between applying your force to the light (empty) cart and applying it to the heavy (loaded with say 2 or 3 kg) cart?

Devise an experiment to compare the two. Describe what you do, and your results.

What do you expect to happen if you apply *twice* the force for 1 second?

Experiment with this with the heavy cart. Describe your results.

Summarize your results below for how force relates to motion and mass. Discuss this with your partners carefully, and with an instructor.

You may have noticed that the force scales use a spring to measure the force. We will use springs a great deal in this course, so you need to know a bit more about how they work.

Guidebook Entry II.8: Spring Forces

Take the silver colored spring and the spring gauge. Measure how much force it takes to stretch the spring by 1 cm, 2 cm, etc. out to 20 cm. Write your results in a table, either in Excel or by hand below.

Graph the force versus stretch.

Extrapolate the graph to predict what length it should be when there is zero force applied to it. How long is this?

How long is the spring when zero force is applied to it? Does this agree with your prediction?

Explain any discrepancy from your prediction, and discuss this with your partners and an instructor.

You can also use the LabPro computer system to create a graph. Hang the spring from the force probe hook. Place the motion detector on the floor immediately beneath the spring. Position the detector such that it can see motion of your hand just beneath the spring. Then place a pencil through the bottom loop of the spring, and use your hand (flat parallel to the ground) to stretch out the spring. Try looking at both force versus time, and distance versus time graphs as you slowly move your hand. Does it do what you expect? Describe.

Now set the vertical scale to force, and the horizontal scale to position. Start data acquisition, and move your hand slowly. Attach a copy of the resulting graph, or sketch below. Does it agree with your graph by hand?

In the final exercise of this session, you are invited to consider the various ways we have applied force and mass.

Guidebook Entry II.9: Force and Mass

Look back over the various ways you have measured or observed force in today's exercise. Which could you do with no motion?

Which measurements or observations required only a very slow and controllable motion?

Which measurements or observations depended on the motion itself to indicate a force was applied?

Describe three different ways that you could measure a force. Would any work in outer space where there is no gravity?

Session 3: Relating Force to Motion

In the last session, you should have discovered that the harder we push on an object, the greater the resulting motion, and that the more massive an object is, the smaller the resulting motion (or equivalently, the greater the force necessary to achieve the same degree of motion). In all of this, we have been deliberately vague about what we mean by motion. In this session, through simultaneous use of the force and motion probes, we will be more exact about what we mean by motion.

In the last session, you also discovered a force law, that is that the force a spring exerts increases linearly with the amount of stretch of the spring. This law is commonly known as Hooke's Law. We will assume that this is familiar in what we are about to do; if it is not familiar, you may wish to look back at your previous activity guide.

Guidebook Entry II.10: Forces on a Hanging Weight

If not running yet, start up your computer and ULI system, and launch Logger Pro. Calibrate your force probe, either with a weight or with the force scale.

Now, hang a 500 g weight from the force probe. What is the force of gravity that is acting on the weight? What direction is that force?

Recall the simple exercise in which you applied 1 N to a cart loaded with 1 kg weights. What happened when you applied a force to the cart that was initially still?

Is the hanging weight moving? [Yes, this is an obvious question.]

Given that force results in motion, what must be the *total* force acting on the weight?

What is applying the additional force?

We can guess that the net force acting on an object is just the sum of individual forces (this turns out to be true, but is not as obvious a fact as it might first appear). If this is true, how big, and in what direction is the second force that is applied to the hanging weight? Draw a sketch of the hanging weight below, and sketch in the forces as arrows.

Does the reading of the force probe agree with your prediction of that second force?

In actuality, we've pulled a little swindle on you. You were asked for the upward force the hook exerted on the weight, but actually the force probe measures the downward force that the weight exerts on the hook. These two forces are equal in size and opposite in direction. In the case of static contact forces, such as we have here, this is easy enough to deduce, or at least to guess. For example, if this wasn't true, you could imagine gluing two pieces of material together, and with unequal forces, the combined object would propel itself along! Or, one would get a different mass for an object depending on what type of weighing pan the balance had, or if one used a piece of weighing paper on the balance to hold the sample. This becomes less obvious (although equally true) for forces like gravity that do not need contact to express themselves. The "equal and opposite" principle is referred to as Newton's Third Law, which we will see again in a couple of weeks.

Guidebook Entry II.11: Forces on a Weight Hanging from a Spring

Now, hang the 500 g weight from a spring, and the spring in turn from the hook on the force probe. Make sure the weight is still. What is the *net* force acting on the weight? How do you know?

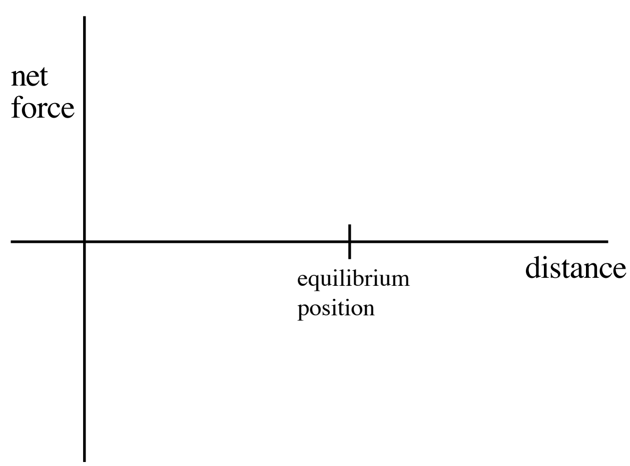
If you didn't write "zero" for the net force in the last question, discuss this with an instructor. Otherwise, continue with this question: What does the force probe read if you pull the weight down 10 cm from its resting point?

What is the net force (exclusive now of the force you are applying with your hand) on the weight when it is 10 cm below its resting point?

What does the force probe read if you push the weight up 10 cm from its resting point?

What is the net force (exclusive of the force you are applying with your hand) on the weight when it is 10 cm above its resting point?

Make a sketch of what you expect the net force to be as a function of position of the weight.



You can make the force probe read the net force by simply subtracting off the extra force necessary to counteract gravity. You could do this by hand, but you can do it automatically by simply stopping the weight at its equilibrium position, then zeroing the force probe (from the Experiment menu). Do this, and now look at the force probe reading at 10 cm above, exactly at, and 10 cm below the equilibrium location. Do the readings agree with your predictions from the previous page?

We have carefully constructed a system in which we can simultaneously measure the position of an object and the net force acting on it. We will now use it to discover the key law for classical motion, Newton's Second Law.

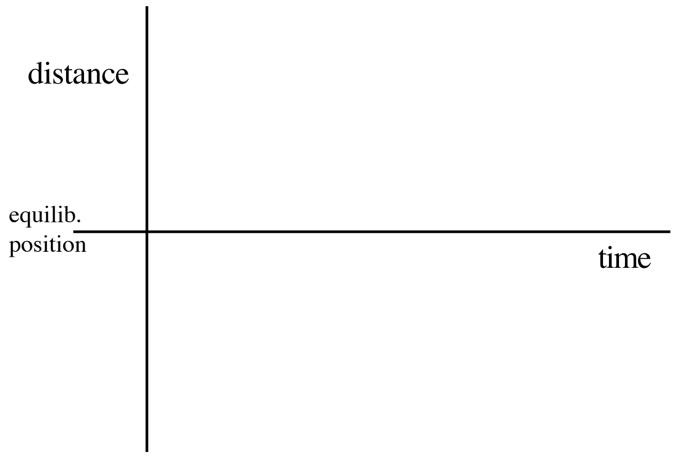
Guidebook Entry II.12: Net Force and Motion

Position the motion detector on the floor underneath the hanging mass. Move the mass slowly up and down 10 cm or so, and make sure that the motion detector is seeing it. If you are having trouble, ask for help!

Check again to make sure that the force probe reads zero at the equilibrium position. Set the vertical axis of Logger Pro to Force, and the horizontal to Distance, and move the weight slowly up and down by hand. Do you get the same force versus distance graph that you predicted on the previous page? Explain any discrepancies.

We are going to look at forces acting on the weight once it is moving, but it is essential that we believe our static measurements (little or no motion) of force also reflect the spring force when the weight is moving. Let's test that. Pull the weight down about 10 cm, and then release it, allowing the weight to oscillate. Do you still get the same force versus distance graph as you did above? Has the force law of the spring been changed at all once it is dynamic, that is, running on its own?

What do you expect the position (or distance) versus time graph for the motion of the freely moving weight on the spring to look like? Sketch it below.



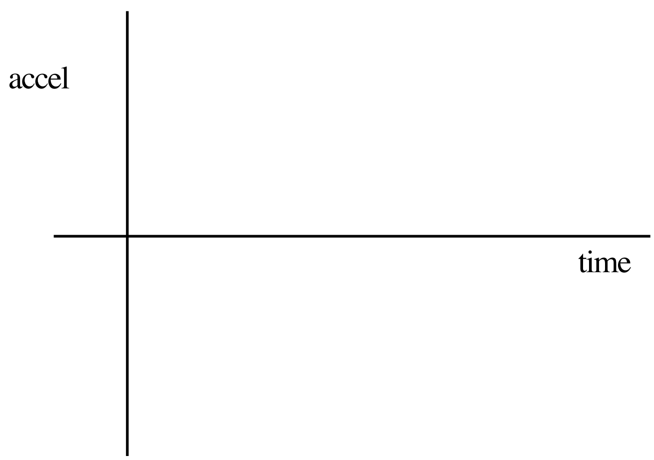
Now look at the position versus time graph in Logger Pro. Does it agree with your prediction?

What sort of mathematical function you have studied looks qualitatively like the graph you see?

Recall that the velocity is the time derivative of the position. Given that, what do you expect the velocity (versus time) function to look like? When should the velocity be large? When should it be zero? Sketch it below, and label some times which correspond to labeled times in your position graph.

Now change the y axis on the Logger Pro graph to velocity (or better yet, choose to show two graphs at once in the Examine Mode turned on in the Analysis menu). Did your predictions come through?

We defined the acceleration to be yet one more time derivative: the derivative of the velocity curve. Given that, predict what the acceleration (versus time) graph should look like by sketching it below. Again, label points that correspond to previous graphs.



Now look at the acceleration graph in Logger Pro. Is it as you predicted? (If your acceleration or your velocity graphs are "noisy" with a lot of jumps an spikes, ask an instructor. You should be able to get reasonably smooth graphs out of this system.)

What we want to find out is how the force relates to the actual motion. We suspect this has something to do with velocity and/or acceleration. Investigate this by looking at two graphs simultaneously in Logger Pro, one being force versus time, the other being either velocity versus time or acceleration versus time. When the force is large, which is large, velocity or acceleration?

When the force is zero, which is zero, velocity or acceleration?

How would you express, as a graph and as an equation, the relationship between force and the appropriate motion variable (i.e. either velocity or acceleration)?

Check your graph prediction by looking at force versus velocity and force versus acceleration graphs in LoggerPro. Sketch what you find below.

You now have at least found the qualitative relationship that force is proportional to acceleration. The final step is to find what the proportionality constant is. You should recall from your dynamics cart work of last session that it depends on the mass, but how? In the next exercise, you will discover that mass dependence.

Guidebook Entry II.13: Mass and Motion

Print a copy of your graph of force versus acceleration. Draw your best line through the data, and find the slope of the line. Give that value below. Be sure to include your units! Ask for help with the units if you are unsure.

Now remove the 500 g weight, and put a 200 g weight on instead. **Rezero the force probe!!!** Produce another graph of force versus acceleration. Print this out, and again extract the proportionality constant.

Does the proportionality constant seem to depend simply on the mass? Write an equation that relates force to acceleration and mass. Discuss this with your partners and an instructor before going on.

In your own words, describe how the unit of the Newton is probably defined.

Now we have the cornerstone of classical mechanics: Newton's second law of F=ma. Much of the next few weeks will be spent simply examining the ramifications of this simple law for a variety of cases.

Incidentally, the Analyze menu in Logger Pro allows you to automatically fit a line (or more exotic functions) to some data--we chose not to have you do that to make sure you knew what the slope of the line is--but it is a good feature of the program to be familiar with!