Unit 10: Introduction to Quantized Waves

In 1905, Albert Einstein made the first bold move into the era of quantum mechanics by proposing that light came in individual packets later called photons. Debate over whether light was a particle or wave went back centuries before Einstein, but Einstein's arguments, having to do with two baffling experiments (the first relating to the color of light emitted by glowing hot objects and the second having to do with electrons ejected from a metal surface by exposure to light), were new and convincing. Each one of these photons has a linear momentum that is related to the wavelength of the light through the relation

$$p = \frac{h}{\lambda}$$

where h is Planck's constant, 6.63 x 10^{-34} Joule seconds. Nineteen years later, Louis de Broglie made the proposal that this same relationship was true for what we considered matter--electrons, protons, even whole atoms--a complete break from classical physics. This expression for the relationship between momentum and wavelength as applied to matter waves is usually referred to as the de Broglie relationship. Each of these major theoretical steps--the photon and matter waves--was rewarded with the Nobel Prize.

Session 1: Adding Quantization: From Light to Photons to Matter Waves

In the last session, you saw that light clearly had wave properties--it exhibited interference properties when it passed through slits. This wave is an oscillation of electric and magnetic fields, called an electromagnetic wave. The electric field is perpendicular to the magnetic field, and both are perpendicular to the direction of propagation. You will learn more about these types of waves if you go on to take more physics. For now, we only need know that these waves travel at 3.0 x 10^8 m/sec, which is usually called the speed of light and given the symbol c.

Guidebook Entry 10.1: Some Practice with Electromagnetic Wave Calculations

Electromagnetic waves come in a wide range of frequencies. For example, the radio waves that we pick up on our FM radios are electromagnetic waves. They are produced by and received through oscillating electric currents in antennas that are pretty large--comparable to the size of a person. I like to listen to WOI-FM, which is broadcast at a frequency of 90.1 MHz. What is the wavelength of this radio wave? How does this compare to what I claim are rough antenna dimensions?

The purpose of making the resonant structure (the antenna) comparable to the wavelength of the resulting electromagnetic radiation is to improve the transmission from the object in resonance (the antenna, or for sound, it might be a guitar string) to the radiating wave (radio waves, sound, or whatever). This is why the low frequency producing double bass in an orchestra is much bigger than the violin or the piccolo. However this rough congruence of size and wavelength does not occur in general—and is strongly violated in radiation from atoms and nuclei. For example, the visible light from a Helium-Neon laser comes from a Helium atom. If an average atom is about 0.2 nm across, how does the He-Ne wavelength of 633 nm compare to this typical atomic size?

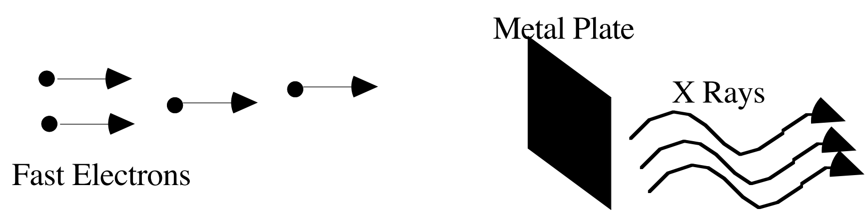
What is the frequency of the He-Ne red light?

Electromagnetic waves can also be emitted from atomic nuclei, which are typically about 5 x 10^{-15} m across. The frequency of these waves, which are called gamma rays, is typically about 2 x 10^{20} Hz. What is the wavelength of such a gamma ray? How does it compare to a typical nucleus size?

What you should have found is that as the sources of electromagnetic waves get smaller, so do the wavelengths, however, the decrease in size is *not* proportional. In fact, the discrepancy in sizes can be factors of a thousand or so! This is a bit surprising; our experience with waves on ropes was that the dimension of the rope to a great extent determined the dominant wavelengths. We also saw diffraction effects were sensitive to the comparative sizes of wavelengths and objects. This is a hint that something funny might be happening here--either light waves don't behave so simply as we might have suspected, or there is something else waving here that determines the frequency (as we found with sound originating from a vibrating string--the string wavelength and the sound wavelength were different--connected only by their common frequency). We will discover a bit of both of these effects are going on. The next section will give more suggestions that something strange is afoot here!

Guidebook Entry 10.2: Making X-Rays

Most all of us have had an X-ray taken at one time or another. An X-ray is like a shadow picture. X-rays are high frequency electromagnetic waves that shine through many materials (just like visible light shines through glass and water, but not steel). A source of X-ray waves is placed on one side of the patient, a piece of film is placed on the other. But have you ever wondered how the X-rays are made? It is really quite simple. Electrons, the same as the electrons swirling around in atoms, or coursing through electrical wiring, are accelerated to a high speed (a significant fraction of the speed of light, in fact) and then they smash into a very dense metal plate. They are accelerated through the vacuum of a vacuum tube by applying a high voltage between their starting point (a hot wire) and the metal plate. In the crash, they emit X-rays.



These X-rays come out with a wide and largely smooth distribution of wavelengths, from *very* long (essentially infinitely long) all the way down to some minimum value which we will call λ_{min}. We also get some total amount of power in the X-ray waves, which we will call P.

What do you guess will happen if we throw electrons at the same speed, but twice as many per second at the metal plate? In particular, how do you think this will affect P?

How do you think doubling the number of electrons will affect λ_{min}?

One could easily do this experiment, although we will not. What one would find is that doubling the *number* of electrons exactly doubles P, and has absolutely no effect on λ_{min}.

Now consider what happens if we keep the *power* of the electrons the same, but we increase their velocity (and therefore the kinetic energy per electron). We can do this easily by increasing the voltage, and decreasing the number of electrons per second proportionally. How do you think this will affect P?

How do you think increasing the electrons' kinetic energy will affect λ_{min}?

What in fact happens is that for this case, we find a small increase in P and a significant decrease in λ_{min}.

It turns out that λ_{min} depends *only* on the electron accelerating voltage. Below is what could have been a set of measurements of λ_{min} as a function of the accelerating voltage.

Voltage	λ_{min}
1000 V	1.44 nm
1500 V	0.96 nm
2000 V	0.72 nm
3000 V	0.48 nm
5000 V	0.29 nm

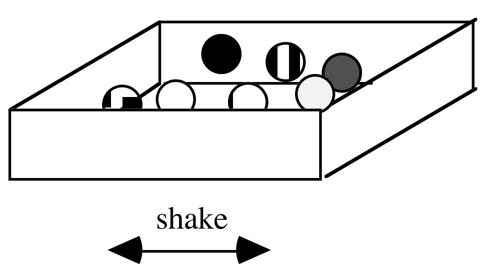
Can you guess a relationship between the voltage (or the kinetic energy of the electrons in electron-volts) and the minimum wavelength for the X-rays? (You may find it easier to find the relationship between voltage and the corresponding *maximum frequency* of X-ray). Check your guess with a graph or some arithmetic, and then with an instructor.

The process of producing X-rays is illuminating (sorry for the pun) because there do not appear to be a lot of resonant frequencies involved. In other words, the stopping process does not excite a "music" in the metal like the fundamental and harmonics of the vibrating string that can explain the minimum wavelength. The X-rays are distributed smoothly over a wide range of wavelengths. This is not to say there are no frequencies that are characteristic of the materials--indeed there are some more intense emissions of certain wavelengths, where those wavelengths depend on the type of material, but the smooth background of X-rays and the minimum wavelength is independent of the type of material. There is some way that the electron kinetic energy gets translated into a wavelength (or equivalently, frequency), but there is no obvious way to do this like there was for a vibrating string of length *l*.

In the last activity, you should have found that the minimum wavelength was inversely proportional to the voltage, suggesting that there is a maximum frequency which is directly proportional to the accelerating voltage. In the next activity, we will turn this around, and see if we can use electromagnetic waves to move electrons, and see if we thereby have limitations on the electron energy.

Guidebook Entry 10.3: The Photoelectric Effect I--A Naive Marbles-in-a-Box Analogy

It was discovered last century that shining light on a piece of metal caused electrons to jump out of the metal. We can imagine these electrons are much like marbles rolling around in the bottom of a box of height Δz open at the top. It is easy for them to roll around the box, but it takes a certain amount of energy ($mg\Delta z$ for our marble analogy) to lift any one of them up high enough to get out of the box. We are going to try to get marbles to hop out of the box by shaking the box back and forth.



The shaking corresponds to the light hitting the metal. We will shake the box back and forth sideways, and there are two different ways we can increase this shaking--by increasing the frequency of our motion (corresponding to the color of the light), or by increasing the amplitude of our motion (corresponding to the brightness of the light).

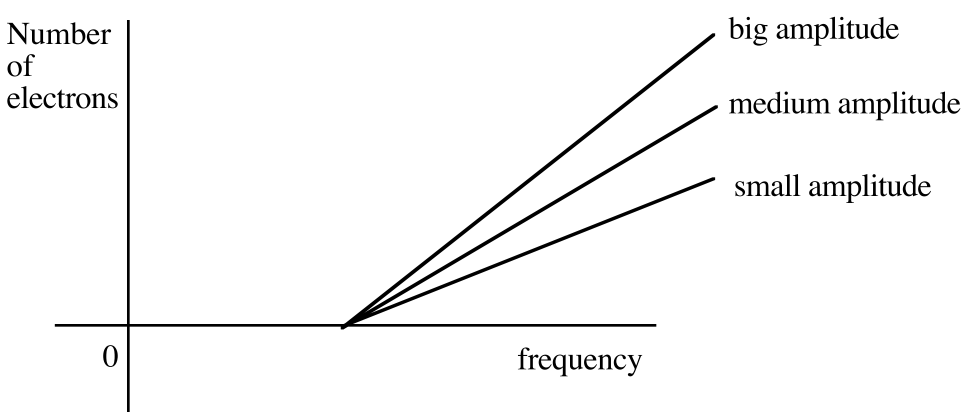
What do you expect to happen to the likelihood of a marble hopping out if we increase the amplitude of our oscillation, keeping the frequency the same?

What do you expect to happen if we increase the frequency of our oscillations, but keep the amplitude of the oscillations the same?

Try this now with our own box filled with balls. What are the effects of increased amplitude (make your variations in amplitude dramatic for best effects!)?

How about differences in frequency? Again, be dramatic!

You should have found that increasing either one of these (amplitude or frequency) would enhance the likelihood of "exciting" a marble out of the box. Similarly, if we scale the amplitude and frequency back, the number of excited marbles decreases. We also would guess that if we cut down on the frequency, we could compensate by increasing the amplitude. However experiment has shown that for electrons in a metal excited by light waves, the dependence on frequency and amplitude is a bit different. As a graph the number of excited electrons versus frequency for two different amplitudes looks qualitatively like

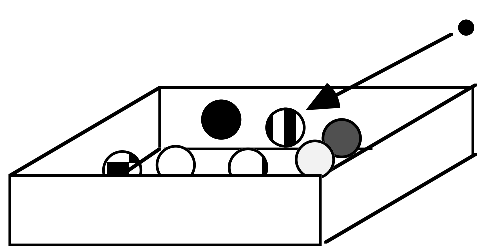


Describe this result in your own words. Would it be surprising for our marbles-in-a-box model? Especially consider that threshold value—the frequency below which *no* electrons are emitted—is that easily explained? It certainly was surprising to physicists around the turn of the century (the turn around 1900, that is!)

Many people felt there must be something peculiar about metal surfaces that caused the photoelectric effect results. However, Einstein made the bold guess that the problem was not with metal surfaces, but with light. He proposed a model in which light has particle-like properties. We'll see how this fixes our difficulties with the photoelectric effect in the next exercise.

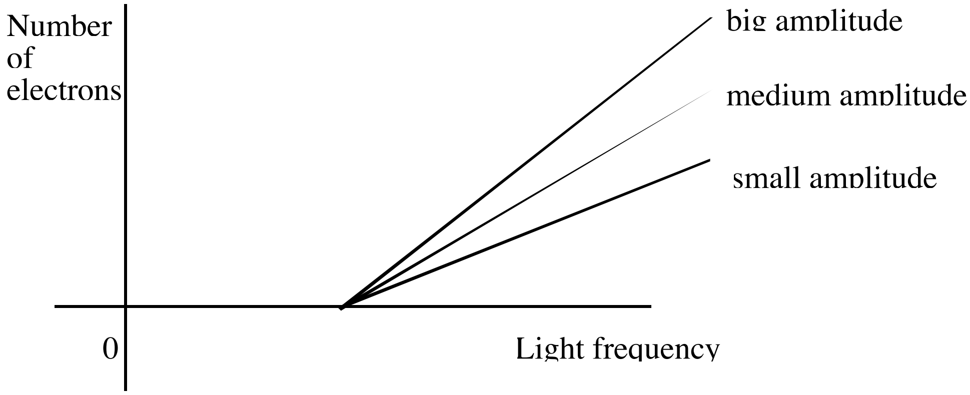
Guidebook Entry 10.4: A Billiard Ball Analogy to the Photoelectric Effect

If we stop thinking of light as waves, but rather as particles, our view of the marble-in-the-box case changes a bit. Now, instead of shaking the box, we are trying to knock the marbles out by hitting them with other, different objects--let's call them pebbles to choose something.



Here you see a black pebble about to hit the vertically striped marble. The black pebble is a particle of light, or a photon. Recall that the height of the wall of the box is Δz; what is the minimum kinetic energy that the photon must have to kick a marble out?

Even if there are many photons coming in, it is still unlikely for two photons to hit the same marble/electron at the same time. If the kinetic energy of the photon corresponds to the frequency through E = hf, can you now explain the graph of electron emission versus frequency, which I have duplicated below? Again, especially address the threshold frequency



It is difficult to communicate clearly what a dramatic conceptual step the photon was. However, even more dramatic was the step taken by Louis deBroglie (the g is pronounced more like a y in his name). In his doctoral thesis, he made the proposition that particles, like electrons or protons, must also have wave properties. The symmetry was too tempting--if what we thought were pure waves (light) also had particle properties (photons with momentum $p = h/\lambda$ and energy $E = hf$), then particles must also have wave properties. The only question was, which was the correct connection to the wave properties--energy or momentum? Since photons appear to have no mass, and electrons clearly *do* have a mass, we can only choose one of these relationships. Louis deBroglie guessed $p = h/\lambda$, and he was correct.

However, if we now say the electron is more properly represented by a wave than by a position is space, and that wave has a wavelength given by the relationship $p = h/\lambda$, we have some new difficulties!

Guidebook Entry 10.5: Momentum and Position:The Heisenberg Uncertainty Principle

Draw a snapshot of a wave below.

You probably showed a sinusoidal wave, extending in principle anyway to + and – infinity. Real objects have some position, and their wave functions must reflect that, and so if this wave represents a particle such as an electron, it in practice must be truncated by chopping off the ends. In that spirit, sketch a waveform that has that same wavelength, but is localized in space.

What is the shortest that a wave function can be (that is, the spatial extent of the wavefunction, which we symbolize as Δx) and still allow us to make some estimate of its wavelength?

If we have a case where we can just barely estimate the wavelength λ, it means the wavelength must have an uncertainty comparable to its size, or in terms of momentum, the uncertainty Δp is approximately equal to p. Given this, what is the product of the uncertainty in momentum (Δp) times the uncertainty in position (Δx)? Use the deBroglie relation, $p = h/\lambda$, to simplify your result.

What you should have found is that the momentum times the minimum spatial extent of the wave function was something around h/2. Presumably if we don't try to localize the electron too much (we make Δx bigger), we can know p better and therefore Δp is smaller. This so called uncertainty principle is a general feature of waves, and a more careful analysis shows that the product is always equal to or greater than $h/4\pi$. This is called the Heisenberg Uncertainty Principle, and is most commonly written as

$$\Delta p \Delta x > h/4\pi$$

where now the Δ refers to the standard deviation of the distribution of either positions or momenta. This is a rather technical definition; most of the time we will be happy to get estimates of these uncertainties within a factor of ten or so--you can sweat the details in later courses if you wish!