Unit 1: Motion and Calculus

It is no accident that Isaac Newton developed both calculus and the study of analytical mechanics simultaneously. He found that the notion of the time rate of change (that is, time derivatives) of velocity and position were naturally related to the motion he observed around him, and therefore developed differential calculus to help describe this motion. He also found the inverse process, equivalent to the summation of these infinitesimal changes, was very useful for applying the laws he developed to complicated systems. This intimate connection between calculus and Newton's laws, however, is not difficult or counterintuitive. While it is important to know some of the calculational tricks of calculus (e.g. the chain rule) and to remember what the derivatives of some common functions are (e.g. derivative of the sine function is the cosine function), the most important relationships are the conceptual relationships between the graphs of the derivative and integral functions. Workshop Physics and the accompanying equipment and software allows us to generate graphs quickly and easily from real motion that you can see and control. In this way, the intuitive connections between the motion, the graphs, and calculus become much more apparent.

First, you must gain some familiarity and confidence in the tools of Workshop Physics. In this unit, you will investigate your own motion using the computers and ultrasonic motion detectors. These devices use the same technique of echo location that bats and many marine mammals use. This unit uses some exercises developed by a group at Tufts University that developed the software and much of the hardware that we use.

Session 1: Graphs of Motion and Velocity

Complete the following two exercises that were developed at Tufts University:

Session 2: Relating Velocity and Distance

In this session, we will investigate in greater detail the relationship between velocity and distance. First, we need to make sure you have a clear notion of the meaning of velocity.

Guidebook Entry 1.1: Driving Along in my Automobile

Imagine that I am driving in my car on Interstate 80 with my cruise control set to maintain my speed at 65 miles per hour. I travel at this constant speed for 2 hours. How far have I traveled? Explain how you got your answer.

Now imagine that my speedometer is broken. I can't tell how fast I am going directly, but I know that I traveled 110 miles in 1.5 hours. Should I worry that I might be arrested for exceeding the speed limit of 65 miles per hour? Explain.

In your own words, describe how velocity is related to time and distance traveled. Then write this as an equation.

Now, imagine that I am in my car waiting for a stop light to change to green so that I can go. Being naturally impatient, I slam the accelerator to the floor as soon as the light turns green, and I begin accelerating rapidly. Twenty seconds later, I am going 60 miles per hour. What would you guess my speed is:

right after the light turned green?

about ten seconds after the light turned green?

This last example shows why physicists define two different velocities: an average velocity, and an instantaneous velocity. Even though we have not defined them, guess what the following are:

The instantaneous velocity at 20 seconds after the light changed green.

The average velocity over the first 20 seconds.

Write down in your own words what you would consider the instantaneous velocity:

and the average velocity:

Make sure to discuss these with your partners and an instructor.

The graphs produced by MacMotion are graphs of instantaneous velocity versus time. In the next exercise, taken again from the Tufts materials, you will be asked to predict velocity graphs from distance graphs, and vice versa.

Finally, we want to be able to calculate a velocity graph directly from a distance graph. We'll use Excel to do this, since it has both graphing and calculation capabilities.

Guidebook Entry 1.2: Calculating Velocity

Let's imagine that we are moving at a constant velocity of 5 m/sec. If I start out initially at position x=0, how far have I traveled in:

1 sec?

2 sec?

5 sec?

t sec?

Write an equation that expresses the distance as a function of time.

Use Excel to graph this function from t=0 to t=10 sec, with 100 entries. Attach a copy of your graph.

What do you expect a graph of velocity versus time would look like? Sketch your guess below:

Now create a third column in your spreadsheet that calculates the velocity over one of the 0.1 sec intervals. That is, the average velocity from 0 to 0.1 sec is the distance traveled in that time divided by 0.1 s. If your first entries are in A2 for time and B2 for distance, this could be entered as =(B3-B2)/(A3-A2) . Graph the distance and calculated velocity columns versus time simultaneously, and attach your graph. Does it agree with your prediction?

You can now use this spreadsheet as a general velocity finding sheet. For example, let's imagine that the position function is given by

$$x = t^2 .$$

Change your second column to reflect this. Sketch what your graph of velocity and time now look like:

Would you call this a graph of average velocity, or instantaneous velocity? Explain. This will need a little discussion with your partners, and you should discuss it with an instructor when you have reached some agreement.

The calculus definition of instantaneous velocity is

$$v = \frac{dx}{dt}$$

where x is the distance and t is the time. Knowing this will allow us to calculate velocity from position functions. In addition, the technique of calculating velocity from spreadsheet values allows us a good way of approximating the velocity function. In fact, MacMotion uses exactly this technique for calculating velocity and acceleration.